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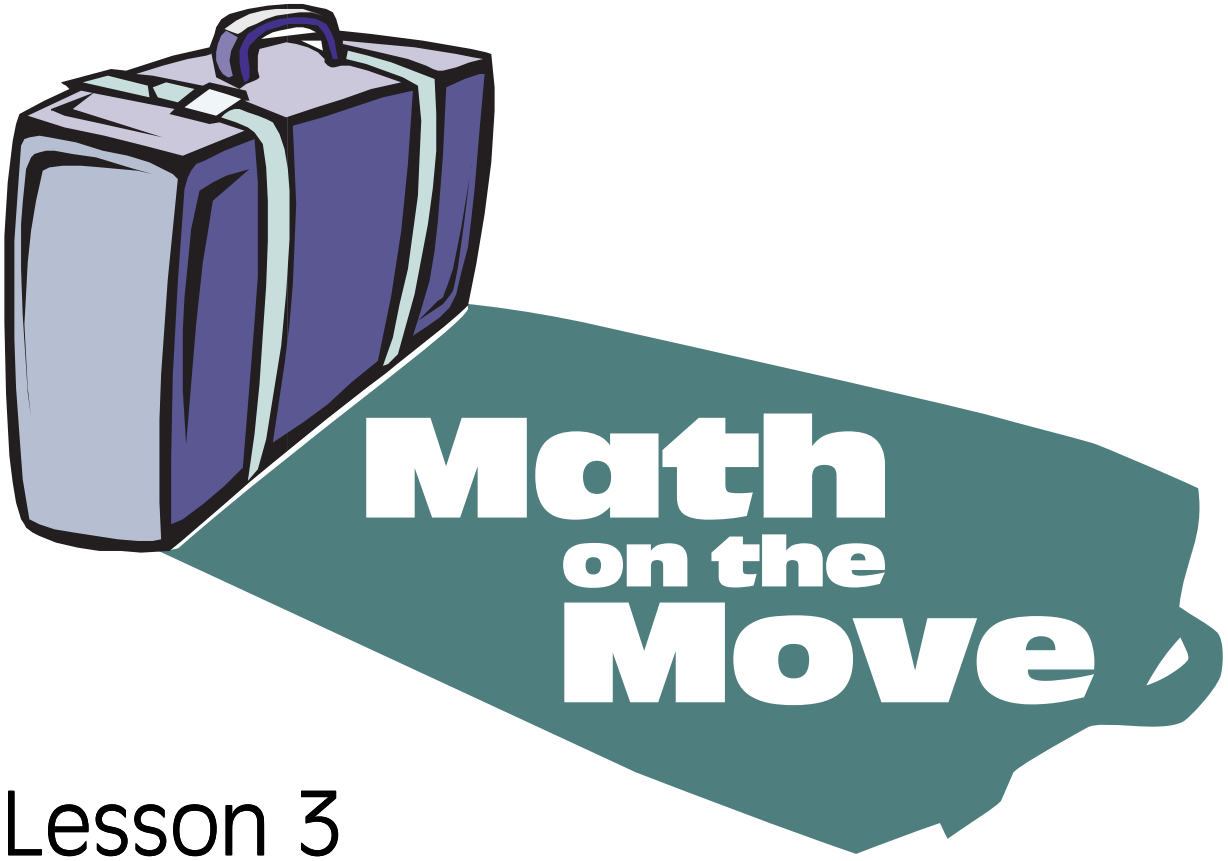

Lesson 3

Multiplying, Dividing, and Exponents

Objectives

- Multiply and divide integers
- Solve word problems that involve operations with integers
- Understand exponents as repeated multiplication
- Evaluate exponents with powers up to three

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Developed by the National PASS Center under the leadership of the National PASS Coordinating Committee with funding from the Region 20 Education Service Center, San Antonio, Texas, as part of the Mathematics Achievement = Success (MAS) Migrant Education Program Consortium Incentive project. In addition, program support from the Opportunities for Success for Out-of-School Youth (OSY) Migrant Education Program Consortium Incentive project under the leadership of the Kansas Migrant Education Program.

You continue to follow the television program on the Iditarod Trail Sled Dog Race.

Example

Rachel continues on with her journey. Still heading out of the Alaskan mountain range, she must travel from the Rainy Pass checkpoint to the Checkpoint in Rohn. When she leaves Rainy Pass, another competitor shares that the temperature is currently $-35°$. After crossing Puntilla Lake, the trail starts climbing upward very fast. Pretty soon, Rachel does not feel tree branches brushing against her. She has climbed above the tree line! When elevation increases so quickly, the temperature drops very fast. Rachel feels that the temperature is now twice as cold as when she left from the Rainy Pass checkpoint.

"What is the temperature now?" you wonder.

Solution

In this problem, there is a lot of *extraneous*, or unnecessary information. We must take only what is important. We are told the temperature is $-35°$ when Rachel leaves. Later, the temperature is twice as cold as $-35°$. Or, in other words, we can write the temperature as

$$\begin{array}{r} -35 \\ \times \quad 2 \\ \hline \end{array}$$

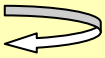
This is the first time we have seen negative numbers with multiplication; however, if we think about the meaning of the statement -35×2, it will give us clues to solving the problem. The statement 2×-35 literally means "two groups of negative 35." We will show this with integer chips.

From before,

there is a positive chip  = +1

and a negative chip.  = -1

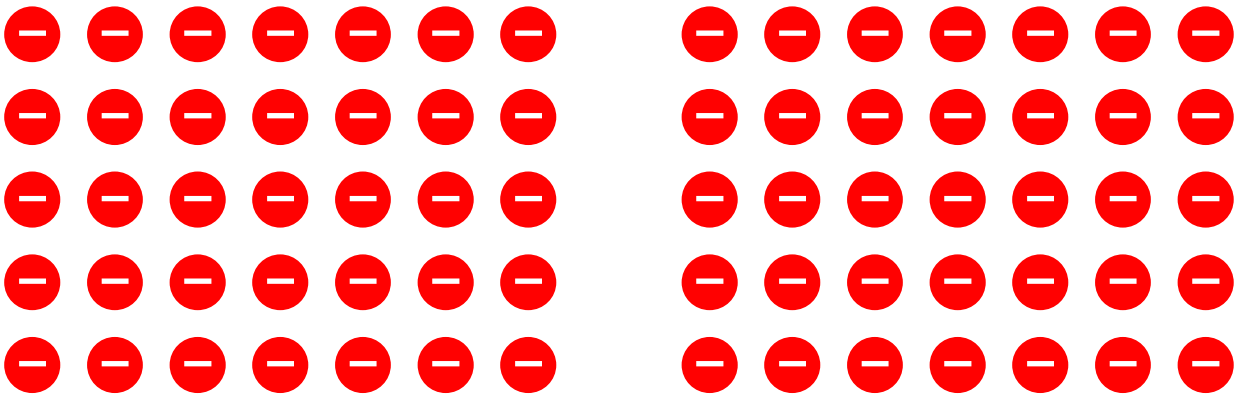
Think Back



We may multiply numbers in any order we want, and it still means the same thing!

$3 \times 2 = 2 \times 3 = 6$

Now we must show two groups of -35. Here they are.



If you count, there are 70 negative integer chips. They represent -70. This means that $-35 \times 2 = -70$. The temperature after Rachel broke the tree line was $-70°$

Just as with addition and subtraction, we don't have to keep using integer chips. There are rules we can use for multiplying and dividing with negative numbers.

Algorithm

To multiply or divide two integers:

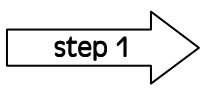
1. Ignore the signs (+ or -) in front of the integers.
2. Multiply or divide as though they were both positive.
3. Write down the product or quotient.
4. Now look back at the signs of the original two numbers.
    a. If they are the same, your answer will be positive (+).
    b. If they are different, your answer will be negative (-).

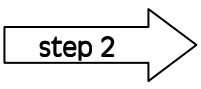
Example

Simplify $12 \div -3$

Solution

First we will set this up as a division problem, and solve it ignoring the signs.

$3\overline{)12}$ 

$\begin{array}{r} 4 \\ 3\overline{)12} \\ 12 \end{array}$ 

$\begin{array}{r} 4 \\ 3\overline{)12} \\ -12 \\ \hline 0 \end{array}$

We end up with 4 and no remainder. Now, we look back to our original numbers, 12 and -3. The signs here are positive (+) and negative (-). These signs are <u>different</u>, so we know the answer will be negative.

Therefore, $12 \div -3 = -4$.

Note that this answer is the same as the answer to $-12 \div 3$.

Example

Find the product. -12×-9

Solution

First we will multiply as we would positive numbers.

$\begin{array}{r} 12 \\ \times \quad 9 \\ \hline \end{array}$ 
$\begin{array}{r} {\scriptstyle 1} \\ 12 \\ \times \quad 9 \\ \hline 108 \end{array}$

Now we look at the signs of the original two numbers, -12 and -9.

They are the same, so our answer will be positive.

Therefore, $-12 \times -9 = 108$.

1. Find the products.

a) -2×-3 b) -4×2 c) 6×5 d) 8×-4

e) -9×-7 f) -6×4 g) 12×-11 h) 8×7

2. Find the quotients.

a) $-8 \div -2$ b) $14 \div -7$ c) $-20 \div 10$ d) $18 \div 9$

e) $-25 \div 5$ f) $24 \div -6$ g) $-100 \div -10$ h) $-2 \div 1$

After a commercial interruption, the Iditarod television show ends. You realize you have been watching TV for over an hour and decide to go outside for some fresh air. As you walk past a cornfield, you overhear the farmer explaining to his workers about corn. He says to them, "Corn grows very quickly. From the time it is one foot tall, it doubles in height each week. You cannot harvest the corn until it is eight feet tall." This gets you thinking.

After the first week, the cornstalks are two feet high. After the second week, the cornstalks are four feet high, and after the third week, the cornstalks are eight feet high. So, after three weeks, the stalks will be tall enough to harvest.

Doubling in height each week means we multiply the height by 2 each week.

$1 \times 2 = 2$ ← Height after one week

$2 \times 2 = 4$ ← Height after two weeks

$4 \times 2 = 8$ ← Height after three weeks

This is the same as repeatedly multiplying by 2 as shown

$$2 \times 2 \times 2 = 8$$

Repeated multiplication can be shown using an **exponent**.

- An **exponent** is a form of repeated multiplication shown with a **base** and a **power**.
  - $2^4 = 2 \times 2 \times 2 \times 2 = 16$, where 2 is the base, and 4 is the power
  - When saying 2^4 aloud, we can say
    "two to the fourth power," or
    "two to the power of four"
  - Whenever a number is raised to the second or third power, it may be read as 4^2, "four squared" or 4^3, "four cubed."

Since the height of cornstalks doubles each week, we can show their height using exponents.

$1 \times 2^1 = 1 \times 2 = 2$ ← Height after one week

$1 \times 2^2 = 1 \times 2 \times 2 = 4$ ← Height after two weeks

$1 \times 2^3 = 1 \times 2 \times 2 \times 2 = 8$ ← Height after three weeks

Example

In a pond there are two fish. Each week the number of fish triples. Using exponents, show how many fish will be in the pond after 5 weeks.

Solution

First, we start with 2 fish. If the number of fish triples, that means we multiply the number of fish each week by 3. By the fifth week, we will have to triple the number of fish five times. In other words, we have to multiply by 3 five times.

$$2\times3\times3\times3\times3\times3$$

Here, we have repeated multiplication. This can be shown using an exponent.

Five 3's

$$2\times3\times3\times3\times3\times3=2\times3^5$$

We can also work backwards when using exponents to change exponential expressions to repeated multiplication.

Example

Determine what the base and power of the exponent 4^6 is, say it out loud, and write it out as a multiplication problem.

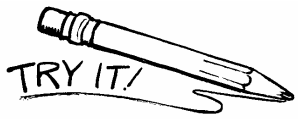
Solution

In the given problem,

$$4^6 \qquad\qquad base^{power}$$

the base is 4 and the power is 6. When read aloud, it should be said as, "four to the sixth power." To write out the multiplication, the base, 4, is the number we multiply, and the power, 6, is the number of times we repeat that multiplication.

$$4^6=4\times4\times4\times4\times4\times4$$



3. Write the following expressions using exponents

a. $3 \times 3 \times 3 \times 3 \times 3$

b. $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$

c. $7 \times 7 \times 7$

d. $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$

4. Say aloud the following expressions, and write them out using the times symbol.

a. 10^3 b. 7^{11}

Now we will get some practice evaluating the exponents.

Example

Evaluate 4^3.

Solution

To evaluate the exponent, first rewrite it as a multiplication problem.

$$4^3 = 4 \times 4 \times 4$$

Now we should multiply the first two terms in the problem.

$$4^3 = 4 \times 4 \times 4$$

$$4^3 = 16 \times 4$$

Bring down the third 4 because we have not multiplied it yet.

Now we can multiply the 4 and the 16.

$$\begin{array}{r} {}^{2} \\ 16 \\ \times 4 \\ \hline 64 \end{array}$$

So, we see the answer is $4^3 = 64$

To evaluate exponents:

1. Rewrite the problem as a multiplication problem.
2. Multiply the first two numbers in the repeated multiplication.
3. Multiply that product by the next number and continue until there are no more numbers to multiply.

5^3

$= 5 \times 5 \times 5$

$= 25 \times 5$

$= 125$

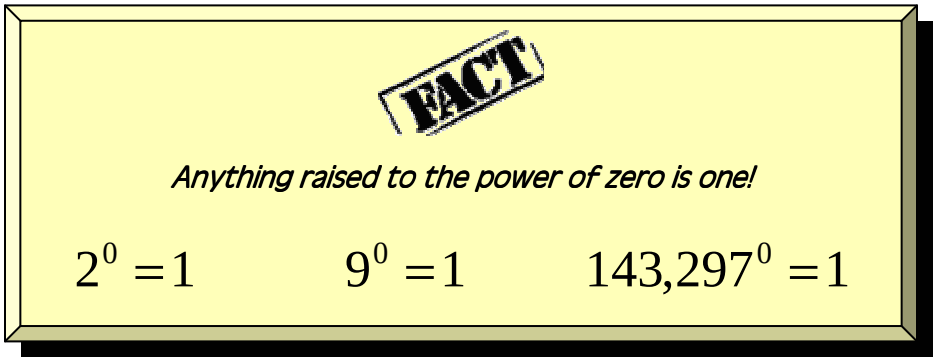
5. Evaluate the following exponents

a. 5^2

b. 3^3

c. 10^1

d. 6^2



☑ Review

1. Highlight the following definitions
    a. exponent
    b. base
    c. power

2. Highlight the Objectives and Algorithm boxes.

3. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems

Math On the Move Lesson 3

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 3, Set A and Set B.

Set A

1. Find the product or quotient.

   a) 3×-7 b) -19×-3 c) -2×9 d) -2×-2

   e) $27 \div -9$ f) $-21 \div -7$ g) $-16 \div 4$ h) $-18 \div -2$

2. Simplify the following exponential expressions.

   a) 2^2 b) 4^2 c) 12^2 d) 7^3

   e) 3^3 f) 8^2 g) 0^3 h) 9^3

Set B

1. Evaluate $(-1)^2$. You should realize that $(-1)^2 = -1 \times -1 = 1$. Now evaluate $(-1)^3$. Keep going. Simplify $(-1)^3, (-1)^4, (-1)^5, \ldots$ until you notice a pattern. Explain the pattern you find.
2. Carlos is a microbiologist, which means he studies very small organisms under a microscope. He gathers the following data about the number of bacteria in a culture over a period of four weeks.

Week	1	2	3	4
Number of Bacteria	10	20	40	80

Fill in the blanks to make each statement true

a) The number of bacteria in week two is $10 \times \_\_\_\_$

b) The number of bacteria in week three is $10 \times 2^{\square}$

c) The number of bacteria in week four is $10 \times 2^{\square}$

d) The number of bacteria in week ten is $10 \times 2^{\square}$

e) The number of bacteria for any week, call it week *n*, is $10 \times 2^{\square}$

3. What is 47829174189^{0}? (*Hint:* Read all the Fact boxes.)

1. a) 6 b) - 8 c) 30 d) - 32
   e) 63 f) - 24 g) - 132 h) 56

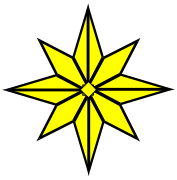
2. a) 4 b) - 2 c) - 2 d) 2
   e) - 5 f) - 4 g) 10 h) - 2

3. a) 3^5 b) 8^{10} c) 7^3 d) 4^{12}

4. a) ten cubed, or ten to the third power, or ten to the power of three
   $= 10 \times 10 \times 10$
   b) seven to the eleventh power, or seven to the power of eleven
   $= 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$

5. a) 25 b) 27 c) 10 d) 36

NOTES



End of Lesson 3