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Math on the Move

Lesson 2

Operations with Integers

Objectives

- Add and subtract integers
- Determine the absolute value of a number
- Solve word problems that involve adding and subtracting integers

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One day you are watching television about the Iditarod Trail Sled Dog Race. The Iditarod is a 1,150 mile trek through Alaska, where each competitor leads a pack of twelve to sixteen dogs through harsh weather and terrain. Also, since the race takes place in Alaska, the competitors ride through temperatures well below zero.

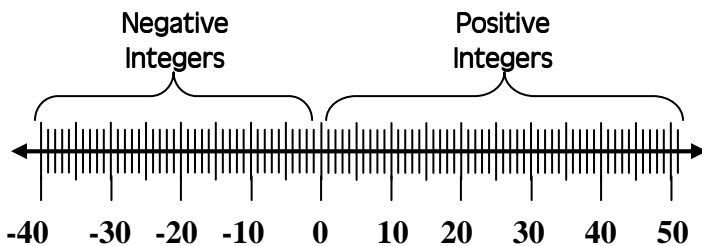
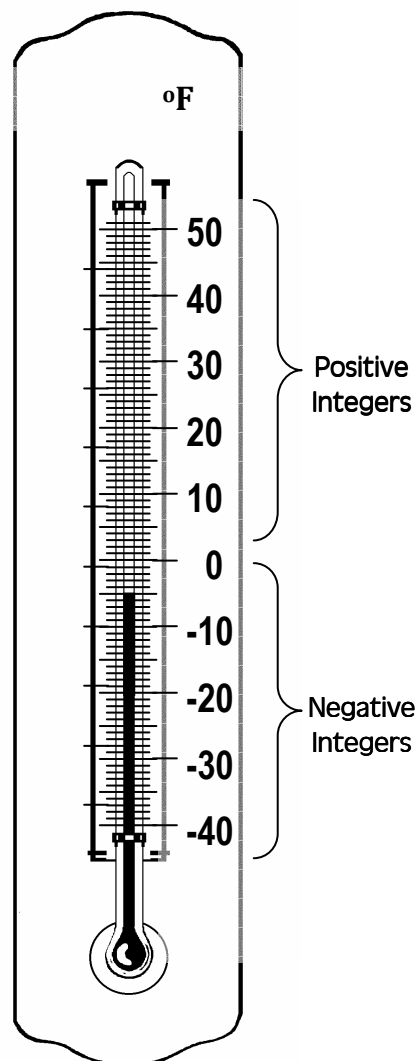
The first concept we must understand is the **integer** system.

- The set of **Integers** is the group of all positive whole numbers, all negative whole numbers, and zero.

... -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

Temperature is one of several real life applications of the integer system. The temperatures that are most familiar to us are usually positive integers. A comfortable spring day is approximately 70° , and water freezes at 32° . Since ice is so cold, it is hard to imagine the temperature going lower than 32° , but the coldest temperature recorded during the Iditarod was approximately -100° .

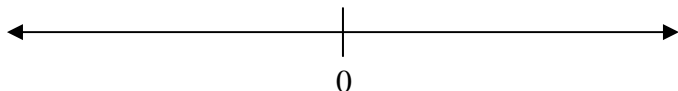
To understand temperature and integers better, examine the thermometer to the right. As you can see the scale on the thermometer goes from -40° to 50° . The negative numbers are all below zero, and the positive numbers are all above zero. This leads us to the idea of zero being neither negative nor positive. Also, we can relate this thermometer to a number line if we turn it on its side.



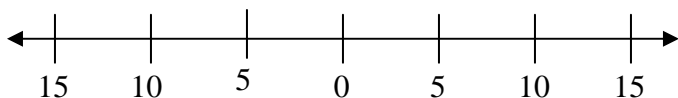
The number line is similar to the thermometer, because it lists numbers in increasing order. However, the numbers increase from left to right instead of bottom to top. The number line helps us remember the order of the integers. It is important to remember that

$$-10 > -20 \text{ and } 10 < 20.$$

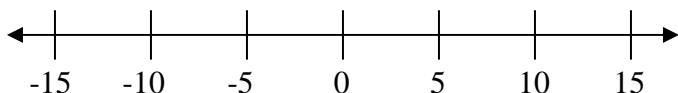
Just remember that the number line has zero in the middle.



Then the numbers increase by a constant interval from zero.



Lastly the numbers on the left of the zero are negative.



FACT

The symbols < or > means less than or greater than. Think of the "<" symbol as an alligator mouth. The alligator mouth always wants to eat the bigger number.

The number line can be a very useful tool for adding and subtracting integers, as you will see.

You find it very admirable that the Iditarod competitors handle such cold temperatures. You have become very intrigued with the competition, especially with contender Rachel Scdoris, who is legally blind. When the race begins you keep track of Rachel’s progress throughout the competition. You find that she feeds her dogs specific nutrients depending on the temperature each day. Since Rachel is blind, she cannot use a thermometer to read the temperature. So, Rachel uses her sense of touch to determine the temperature outside. In other words, Rachel can approximate the temperature by determining if it is colder or warmer than the day before.

Rachel feeds her Dogs nutrients based on the chart given below.

Temperature Range (Fahrenheit)	less than -36°	-35° to -16°	-15° to -1°	0° to 14°	more than 15°
Dog Food	Nutrient A	Nutrient B	Nutrient C	Nutrient D	Nutrient E

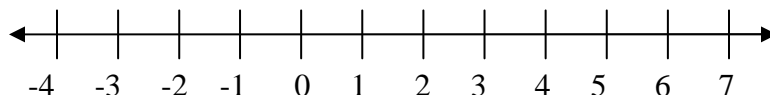
When Rachel starts the competition, she is told that it is currently 7 degrees Fahrenheit. Based on this information, she feeds her sled dogs Nutrient D . From this point on, Rachel must use her senses to determine the temperature for the rest of the competition.

Example

On day two of the competition, Rachel feels as if the temperature has dropped 10 degrees from the 7 degree temperature. What is the current temperature outside, and which nutrient should she feed her dogs?

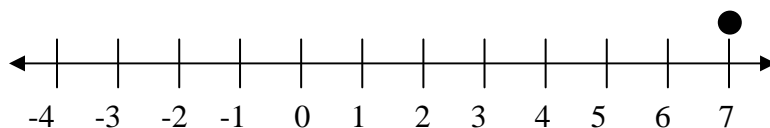
Solution

To answer this problem, we will use the number-line-method. On the first day of the journey, the temperature was 7 degrees. Also, because the temperature dropped on the second day, we will draw a number line that includes the number 7, as well as the integers less than 7.

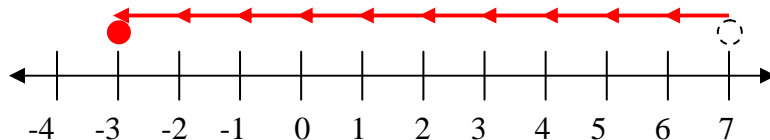


**Note:* The number line does not have to center around the number zero. Zero simply separates the positive and negative integers.

Next, we place a dot above the number seven on the number line. It represents the temperature on the first day.



Then, we must move the dot 10 integers to the left on the number line, because Rachel said she felt the temperature drop 10 degrees.



This means the temperature on the second day is -3° . Next, we refer back to the Nutrition chart to see which nutrient Rachel should feed her dogs. Since -3° is between -15° and -1° , Rachel should feed her dogs Nutrient C.

Even though we solved this problem using a number line, it is really a subtraction problem. On day one it was 7 degrees, and on day two it was 10 degrees less than day one.

$$7 - 10 = -3$$

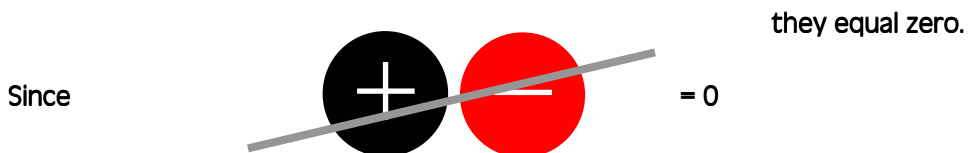
Thus, it was -3 degrees.

Solving a problem in this way is called the number-line method.

Notice that when using the number-line method, We put a dot on the first number in the problem, in this case, 7. Then, if we are subtracting, we move to the left. If we are adding, we move to the right. In this case, we moved 10 integers left, and ended with -3.

Another way to solve this problem is called the integer chip method.

There are two different types of integer chips,



we say that they “cancel out.” To show the cancellation, we cross them out whenever we see them together.

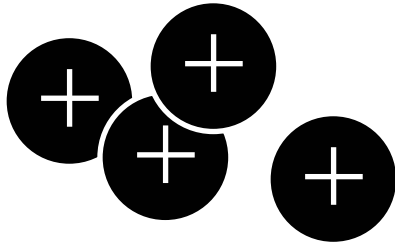
Now let's try a similar temperature problem using this method

Example

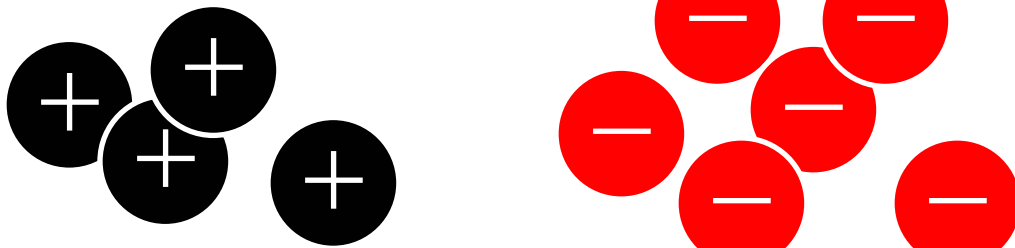
The temperature is 4 degrees, and it drops 6 degrees. What temperature is it now?

Solution

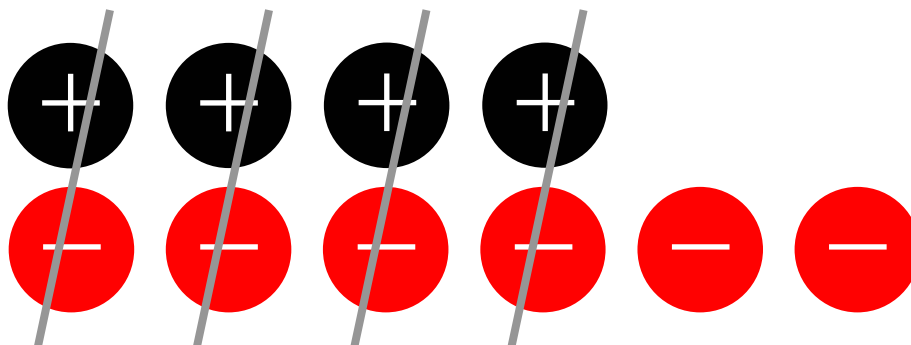
We started at positive four degrees, so we show this with 4 positive chips.



Then, since the temperature drops 6 degrees, we will show this by adding six negative chips to the pile.



Now, recall that we said $\oplus \ominus = 0$, and that it cancels out. If we regroup the chips a little, we see that we can cancel some out.



Finally, we are left with two negative chips, so the temperature will be -2 degrees.

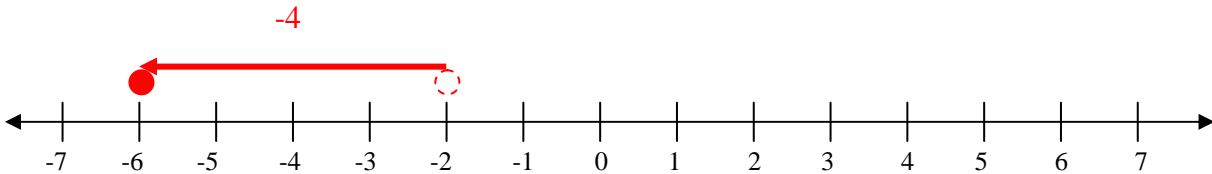
Let's look at another example.

Example

What is $-2 - 4$?

Solution

First we will use the number-line method. We have a negative number minus another number. Because of this, we will be moving in the negative direction (left). First, start at negative two on the number line; this is our starting position in the expression. Next, move four spaces to the left.



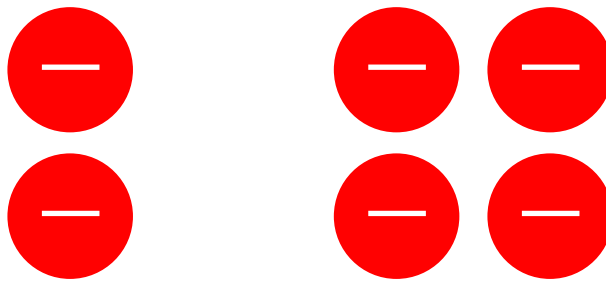
We end up at -6 , so our answer is $-2 - 4 = -6$

Using integer chips, we should get the same answer.

There are actually two ways to think about this problem, and we can show both with the chips.

We can think about $-2 - 4$ is two negative chips, plus four more negative chips,

Or $-2 + -4$



We see that there are 6 negative chips, so our answer is -6 .


Example

On the third day of the journey, Rachel heads into the heart of the Alaskan mountain range. She feels that the temperature has dropped 40 degrees from what it was on the second day. What is the temperature on the third day, and which nutrient should she feed her dogs?

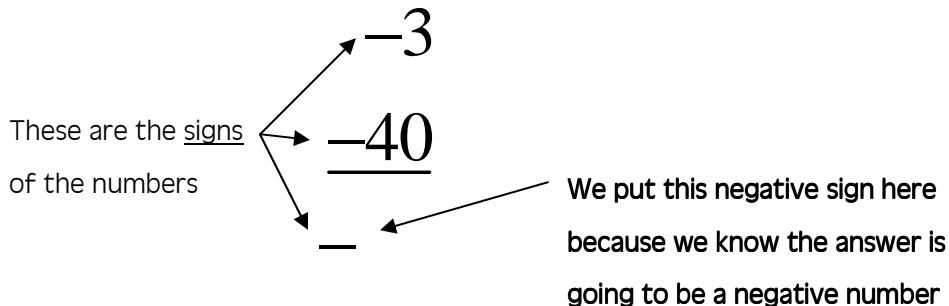
Solution

To solve this, it would be difficult to use a number line with 40 integers on it. Also, keeping count of this many integer chips often leads to mistakes. For this problem, we will use a shortcut. We will rewrite this as a subtraction problem.

On day two it was -3 degrees, and on day three it was 40 degrees less than day two.


$$-3 - 40 =$$

Looking at the problem this way, it may not be clear what the solution is. We will use an algorithm to help us find the solution. First, notice that the initial temperature is a negative number (-3), and we are subtracting a number from that. This means our solution will also be negative. Let's rearrange our problem to look more like this.



These are the signs of the numbers

-3
-40
_

We put this negative sign here because we know the answer is going to be a negative number

When we rewrite the problem this way, we can use the following rule.

“If signs are the same, keep the sign and add the **absolute values** of the numbers.”

- A number's distance from zero is its **absolute value**. To show that you are taking the absolute value of a number, put long vertical lines to the left and right of the number.

For instance, to show the absolute value of -12 , we say $|-12| = 12$.

- The absolute value of a number is always the positive value of that number.

For example, $|-5| = 5$ and $|7| = 7$

Now, we look at our original problem: $-3 - 40$. The signs are the same, so we will add the absolute values of the numbers. First we must find the absolute values.

$$\begin{array}{l} -3 \longrightarrow |-3| = 3 \\ \underline{-40} \longrightarrow |-40| = 40 \\ - \end{array}$$

Then we add the absolute values.


$$\begin{array}{l} |-3| = 3 \\ |-40| = 40 \end{array} \quad 3 + 40 = 43$$

Lastly, we keep the similar sign of the numbers.

$$\begin{array}{l} -3 \\ \underline{-40} \\ -43 \end{array}$$

So, on the third day it was -43° . Therefore, Rachel must feed her dogs Nutrient A.

Here is a simpler form of the previous method.

 Algorithm	
To add two integers:	$-7 + 3$
1. Ignore the signs (+ or -) of the numbers. (This is taking the absolute value.)	$7 \quad 3$
2. Find the bigger number.	$\textcircled{7} \quad 3$
3. Take the sign that was in front of the bigger number. This will be the sign of the answer.	Use $-$.
4. a) If the signs of the two numbers were the same, add the two numbers. b) If the signs of the two numbers were different, subtract the two numbers.	Different, so $7 - 3 = 4$
5. Write the sum or difference next to the sign found in step 3.	Answer is -4 .

Example

On the fourth day of her journey, Rachel is leaving the Alaskan Mountain range. She feels that the temperature has risen 15 degrees from what it was on the third day, -43° . What is the temperature on the fourth day, and which nutrient should she feed her dogs?

Solution

We must first understand the problem. It says that the temperature is rising 15° from -43° . Therefore, we must find

$$-43 + 15$$

This problem has negative integers in it, so:

Step 1: Ignore the signs.

$$-43 + 15$$

Step 2: 43 is bigger than 15.

Step 3: Because 43 is the larger number, we will use the sign in front of the 43 (the "-" sign) in our answer.

Step 4: We notice the signs of the two numbers are different, so we will find the difference by subtracting the numbers.

$$\begin{array}{r} 3 \ 13 \\ 4 \ 3 \\ - 1 \ 5 \\ \hline 2 \ 8 \end{array}$$

Step 5: Now we will return to Step 3 and remember we must put a minus sign "-" in front of the 28.

$$-28$$

So the temperature was -28° on the fourth day. Referring to our chart,

Temperature Range (Fahrenheit)	less than -36°	-35° to -16°	-15° to -1°	0° to 14°	more than 15°
Dog Food	Nutrient A	Nutrient B	Nutrient C	Nutrient D	Nutrient E

Notice that -28 is between -35 and -16 . Because of this, she must feed her dogs Nutrient B.



1. Draw integer chips to represent the following problems, then show the cancellation (if there is any), and write the sum or difference as a number.

a) $3 - 7$

b) $-8 - 2$

2. Plot the following values on the number line and label each point with the appropriate letter.

a) 5

b) 8

c) -3

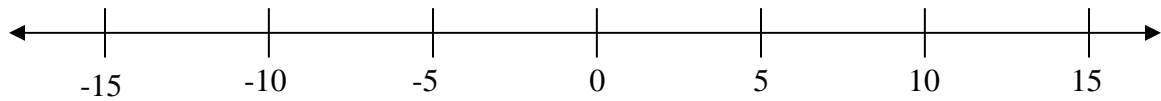
d) -6

e) 0

f) -1

g) -16

h) $|-2|$



3. Find the absolute values.

a) $|-5|$

b) $|-10|$

c) $|13|$

d) $|4|$

4. Simplify each of the following using the algorithm (step-by-step) method from before.

a) $13 - 7$

b) $-4 - 2$

c) $-27 + 17$

d) $-24 - 1$

e) $-10 + 3$

f) $3 - 8$

5. If the temperature is -13° Monday, and the weatherman predicts it will rise 15° overnight, what do you expect the temperature to be in the morning?



Review

1. Highlight the following definitions:

a. integers

b. absolute value

2. Highlight the Algorithm Box.

3. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.



Practice Problems

Math On the Move Lesson 2

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 2, Set A and Set B.

Set A

1. The Buffalo Bills had a 15 yard gain, and then an 18 yard loss. Represent this with integers, then state the total number of yards gained or lost.
2. In a single day, the stock for Ford went down 12 points, then went up 3 points. Use integers to show the total change of the stock for the day.
3. Earl owns a factory where he makes milk crates. It costs \$.35 to make one crate, and he sells one crate for \$.50. If he makes 1000 crates, and only sells 500 of them, use integers to represent Earl's total loss (-) or gain (+). (*Hint:* Think of the cost as a negative value, because that is how much Earl spends.)
4. Find the sum or difference.
 - a) $3 - 9$
 - b) $-19 - 3$
 - c) $-2 + 9$
 - d) $-2 - 2$
 - e) $27 - 9$
 - f) $-21 - 7$
 - g) $-16 + 4$
 - h) $-18 + 2$

Set B

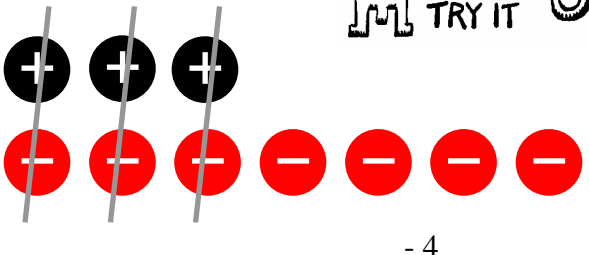
1. Using the definition of absolute value, when simplified, will the expression $\left| \frac{-\sqrt{3^3 - 7} + 18}{40} \right|$ be positive or negative? Why?

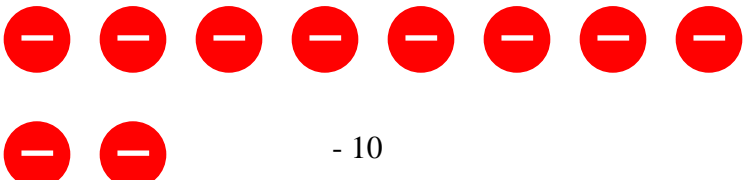
Math On the Move

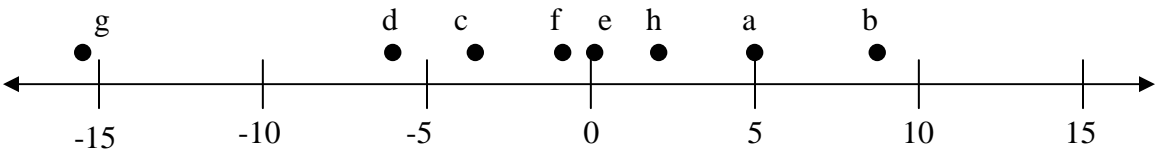
2. Write and solve a real world example that involves subtracting a negative number.

3. Carlos received a paycheck for \$300 from his job, and found \$20 on the way home from the dentist. The dentist appointment cost him \$25 in co-payment. When he got home, Carlos opened his mail. One letter said that his distant uncle, Uncle Jesús, died unexpectedly, and left him \$1500. Another said that his rent for the month was due, for a total of \$400. That night, Carlos tried to start his car, but it made an awful noise. When he took it to the mechanic, he discovered that there was a crack in his head gasket. It will cost him \$950 to repair the head gasket. Does Carlos have enough money to pay his debts? If not, how much does he need. If yes, how much will he have left?

ANSWERS TO
TRY IT

1. a) 

b) 

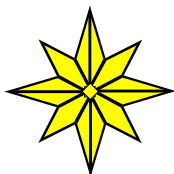
2. 

3. a) 5 b) 10 c) 13 d) 4

4. a) 6 b) - 6 c) - 10 d) - 25 e) - 7 f) - 5

5. 2°

NOTES



End of Lesson 2