Student Name: _____	Contact Person Name: _____
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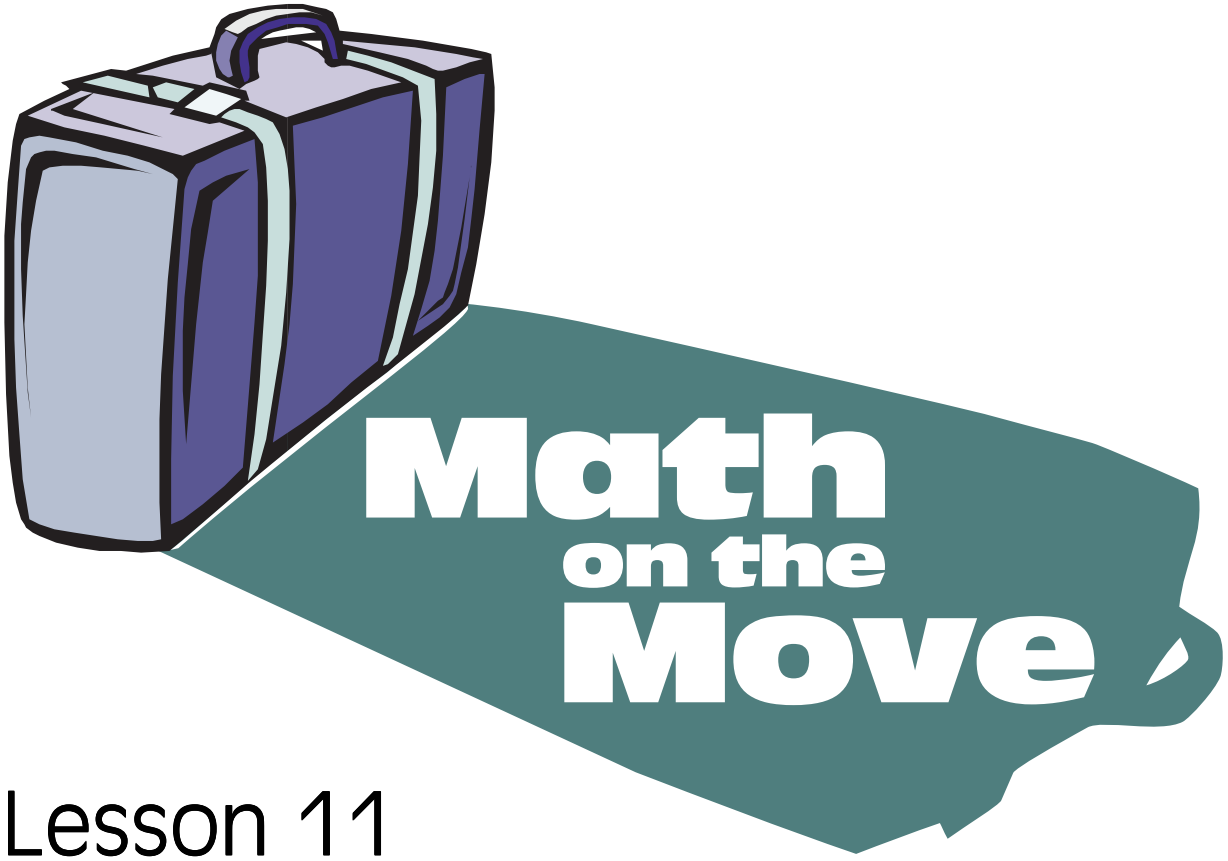

Lesson 11

Variables and Unknowns

Objectives

- Understand how variables are used in math
- Solve simple equations and check with substitution

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Try to fill in the blank.

$$3+\_\_=5$$

It shouldn't have been difficult to see that the answer is 2. What about this one?

$$12-\_\_=7$$

This one was a little harder, but maybe you still found the answer, 5, since 12 – 5 = 7. Okay, here's a harder one.

$$\_\_\div 6=1.5$$

We can solve all these equations using **Algebra**.

- **Algebra** is a branch of mathematics that uses letters or symbols to represent unknown numbers.

  For example, 3 + ? = 8 could be written as $3+n=8$, where "*n*" represents the unknown number.

Algebra is an extremely useful tool. It has helped mathematicians and scientists solve many problems, including the flight path of the first rocket sent to the moon, as well as Einstein's famous,

$$E=mc^2.$$

In English, this equation is read, "energy equals mass times the speed of light squared." The letters all stand for something else. The E, the m, and the c are all examples of **variables**.

- A **variable**, also called an **unknown**, is a symbol (almost always from the English alphabet) that is used in place of a number.

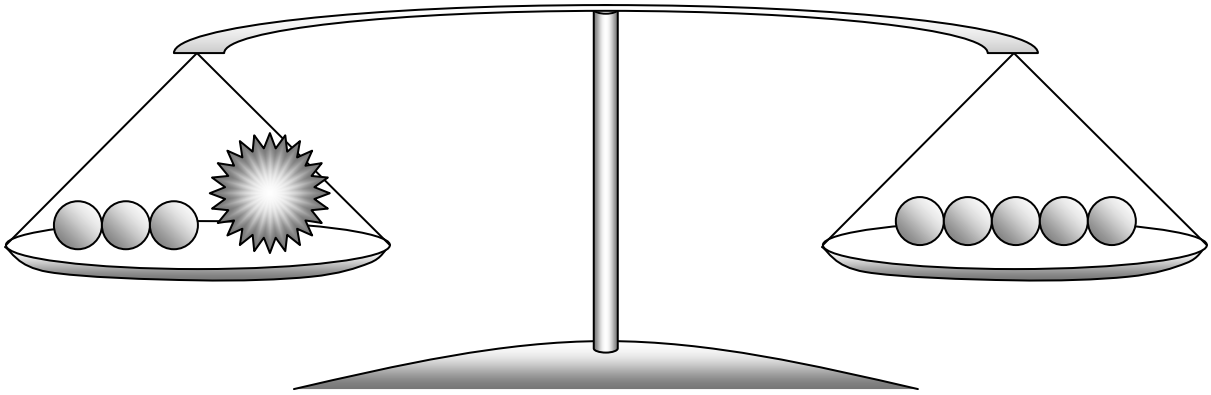
Let's back up for a moment, so we can understand this a little more. Here is the first question asked in the lesson.

$$3+\_\_=5$$

There is one number that is not known, the one in the blank. For this unknown, we will introduce a variable. Why not use *x*? Now this equation reads,

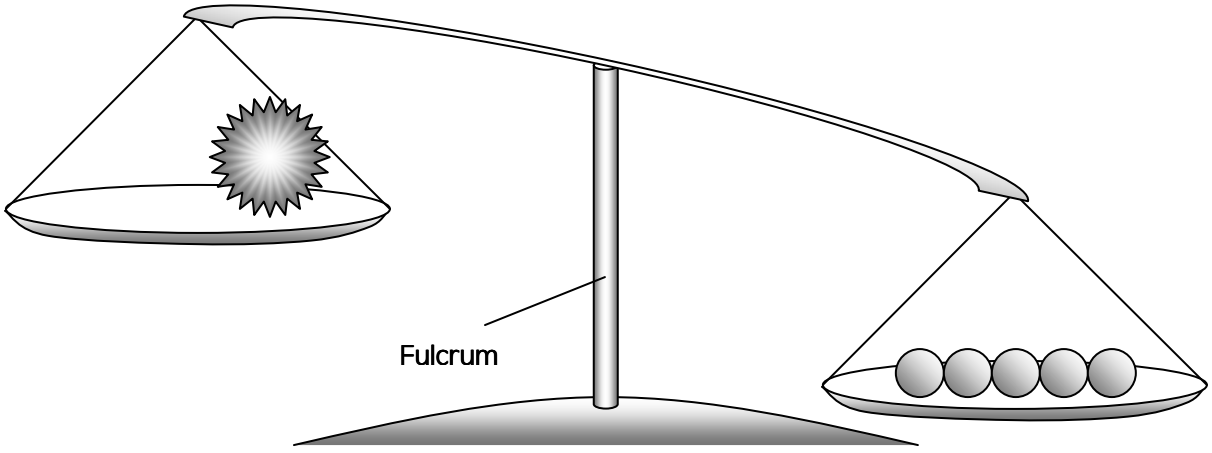
$$3 + x = 5$$

Imagine this equation as some objects on a balanced scale.

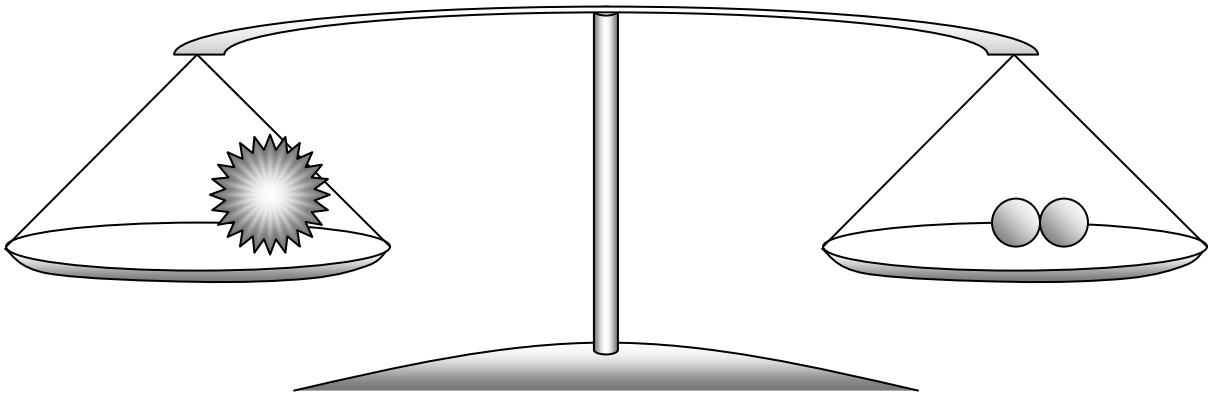


$$3 + x \qquad = \qquad 5$$

In order to solve this, we need to get the fancy stone (the *x*) by itself on the scale, and still have the scales balance. That's easy, let's just take the three round stones off the left pan.



The scales aren't balanced now! Since we took three round stones from the left, we probably should have taken three stones from the right as well. By removing three stones from the right pan, we see,



$$x = 2$$

Let's do a similar problem, but with numbers and variables this time.

Example

Solve for *n*. $n+8=22$

Solution

$$\begin{array}{rcl} n+8 & = & 22 \\ -8 & & -8 \\ n & = & 14 \end{array}$$

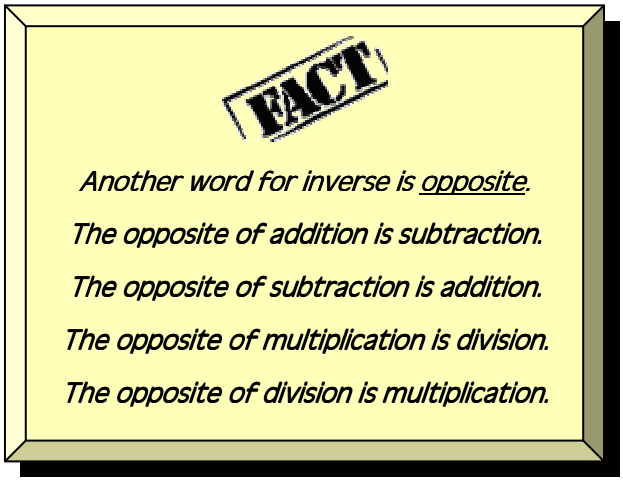
These cancel.

Rewrite the equation, and draw two vertical lines down the equal sign. These represent the fulcrum of the balance scale.

In order to solve for any variable – in this case "*n*" – you need to get the variable by itself. In this equation, 8 is being added to "*n*". We can get rid of the 8 by subtracting it from "*n*". But, in any equation, whatever operation is done on one side of the = sign must be done on the other side. We must subtract 8 from 22, in order to keep both sides of the equation equal.

Notice that in both examples, with the balance scale and with numbers, when a number was being added to our variable, we got rid of it by subtracting. Addition and subtraction are called **inverse operations**.

- An **inverse operation** is an operation that undoes another operation.
  - Addition and subtraction are inverses.
  - Multiplication and division are inverses.



Example

Solve for *h*. $h - 13 = 4$

Solution

First, we will rewrite our equation with lines on either side of the equal sign.

$$h - 13 \mid = \mid 4$$

Now, whenever we write the "=" sign, we'll be sure to keep it between the lines.

Next, we will do the opposite of subtracting 13, which is adding 13. We will do this to both sides of the equation.

$$\begin{array}{r|c|l} h - 13 & = & 4 \\ +13 & & +13 \end{array}$$

The next step is to simplify both sides of the equation.

$$\begin{array}{rcl} h - 13 & = & 4 \\ +13 & & +13 \\ \hline & & +17 \end{array}$$

Lastly, we write our final answer.

$$\begin{array}{rcl} h - 13 & = & 4 \\ +13 & & +13 \\ \hline & & 17 \end{array}$$

$$h = 17$$

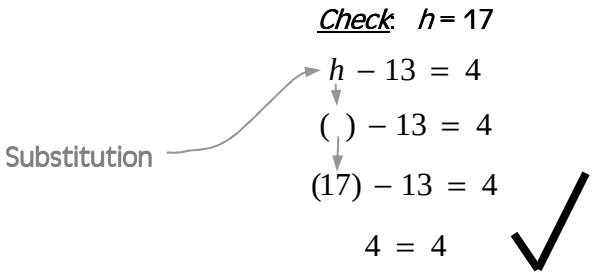
Believe it or not, we can actually prove our answer is correct using a technique called **substitution**.

- **Substitution** is the term used to describe something being replaced by something else of equal value.

  For example, if $x = 3$, then $1 + x = 1 + 3$.

Look to the previous example. We were asked to solve for *h* given the equation $h - 13 = 4$.

We determined $h = 17$. To <u>check</u> our answer, we will <u>substitute</u> the number 17 in place of the variable *h*.



Write the answer you want to check.

Rewrite the original equation.

Substitute your answer.

Simplify with arithmetic.

If the numbers at the end are equal, we say it "checks," and we draw a check-mark.

1. Solve for the variable in each equation, and check to show they are correct.

a) $y + 5 = 9$

b) $x - 3 = 12$

c) $4 + a = 19$

d) $f + 20 = 57$

e) $c - 4 = 27$

f) $17 = k + 4$

g) $-3 = c + 19$

h) $42 = t - 2$

i) $r - 19 = 8$

Example

Solve for *k*. $4k = 28$

Solution

Since the 4 and the *k* are being multiplied, we know we must divide both sides by 4 in order to get *k* alone. We show division by making a fraction.

$$\frac{4k}{4} \Big| = \Big| \frac{28}{4}$$

> 
>
> *When numbers are next to variables with no sign in between them, it means they are being multiplied.*
>
> $4k = 4 \bullet k$, just as $-12w = -12 \bullet w$

Next, we will cancel the fours on the left side and divide 28 by 4 on the right side of the equal sign to solve for *k*.

$$\frac{\cancel{4}k}{\cancel{4}} \Big| = \Big| \frac{28}{4}$$

$$k \Big| = \Big| 7$$

Finally, we check our answer.

Check: $k = 7$

$4k = 28$

$4(\ ) = 28$

$4(7) = 28$

$28 = 28$ ✓

Example

Solve for *w*. $-8w = 104$

Solution

$$\frac{-8w}{-8} \Big| = \Big| \frac{104}{-8}$$

$$w \Big| = \Big| -13$$

Check: $w = -13$

$-8w = 104$

$-8(\ ) = 104$

$-8(-13) = 104$

$104 = 104$ ✓

Think Back

- *When multiplying or dividing two integers, if they have the same signs (+, + or -, -), the answer will be positive.*
- *If the signs are different (+, - or -, +), the answer will be negative.*

Example

Solve for *p*. $\frac{p}{4} = 6$

Solution

Recall that $\frac{p}{4} = p \div 4$ so we must do the opposite of division and multiply each side by 4.

$$4 \bullet \frac{p}{4} = 6 \bullet 4$$
$$p = 24$$

Check: *p* = 24

$$\frac{p}{4} = 6$$
$$\frac{(\ )}{4} = 6$$
$$\frac{(24)}{4} = 6$$
$$6 = 6 \quad \checkmark$$

2. Solve for the variable in each equation, and check to show they are correct.

a) $2a = 14$

b) $\frac{x}{3} = 11$

c) $6s = 96$

d) $9 = \frac{m}{4}$

e) $8y = 96$

f) $25z = 100$

g) $\frac{h}{5} = 5$

h) $\frac{x}{4} = 8$

i) $13t = 130$

☑ Review

1. Highlight the following definitions
    a. algebra
    b. variable
    c. unknown
    d. inverse operation
    e. substitution

2. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems

Math On the Move Lesson 11

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 11, Set A and Set B.

Set A

1. Solve for the variable, and check your answer.

a) $4x = 12$ b) $n + 1 = 7$ c) $14 = z - 5$

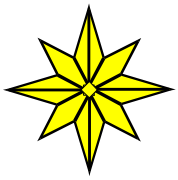
d) $\frac{r}{6} = 18$ e) $x - 80 = 120$ f) $64 = 8w$

g) $z + 80 = 12$ h) $y + 4 = -7$ i) $\frac{t}{3} = -19$

Set B

1. Remember the third fill-in-the-blank problem? It was $\_\_ \div 6 = 1.5$. Solve for the blank, using algebra. This means you will have to pick out a variable to use and solve for it. You can even prove if you were right by checking!
2. Recall from the lesson that we do the inverse operation to undo something. For instance, in $3x$, we would undo the 3 by dividing. What would we do to change $\sqrt{x}$ into x?

1. a) $y = 4$ b) $x = 15$ c) $a = 15$ d) $f = 37$
   e) $c = 31$ f) $k = 13$ g) $-22 = c$ h) $t = 44$
   i) $r = 27$
2. a) $a = 7$ b) $x = 33$ c) $s = 16$ d) $36 = m$
   e) $y = 12$ f) $z = 4$ g) $h = 25$ h) $x = 32$
   i) $t = 10$



End of Lesson 11