

Topic 1

Basic Geometry and Triangles

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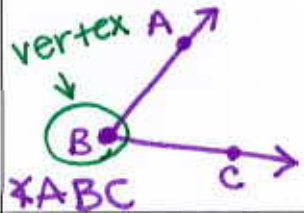
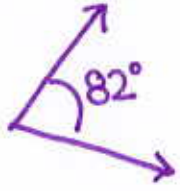



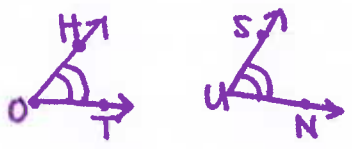
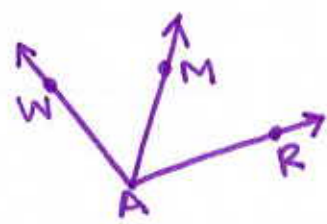
Triangles


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Vocabulary Word	Definition	Picture
Angle	Formed by two rays that meet at a common point.(vertex)	 <p>vertex A B C ∠ABC</p>
Acute Angle	An angle whose measure is between 0° and 90° (not = to 0° or 90°)	 <p>82°</p>
Obtuse Angle	an angle whose measure is between 90° and 180° (not = to 90° or 180°)	 <p>125°</p>
Right Angle	an angle whose measure is 90°	 <p>90°</p> <p>* we use a □ to show 90°</p>  <p>90°</p>
Congruent Angles	Two or more angles that have the same degree measure (symbol → ≅)	 <p>∠HOT ≅ ∠SUN</p>
Adjacent Angles	Two angles who are "next to" each other sharing a common side and vertex.	 <p>∠WAM is adjacent to ∠RAM</p>

ray


Note: When naming angles, the vertex is always the middle letter

∠WAM is adjacent to ∠RAM

Vocabulary Word	Definition	Picture
<p>Angle Bisector creates 2 \cong angles</p>	<p>a ray that splits an angle into two congruent angles. (Bisect means cut in half)</p>	
<p>Supplementary Angles supplementary</p>	<p>two or more angles whose sum is 180°</p>	
<p>Complementary Angles complementary</p>	<p>two or more angles whose sum is 90°</p>	
<p>Vertical Angles</p>	<ul style="list-style-type: none"> * formed by the intersection of two lines. * across from each other * vertical angles are \cong 	
<p>Linear Pair</p>	<p>a pair of adjacent, supplementary angles (that form a straight line)</p>	

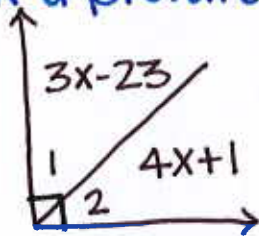
Examples

add to 90°

measure of 41

- 1) Angles 1 and 2 are complementary. If $m\angle 1 = 3x - 23$ and $m\angle 2 = 4x + 1$, find the value of x and the measures of the angles.

Draw a picture:



Angles:

$$m\angle 1 = 3(16) - 23 = 25^\circ$$

$$m\angle 2 = 4(16) + 1 = 65^\circ$$

Write equation (and solve)

$$3x - 23 + 4x + 1 = 90$$

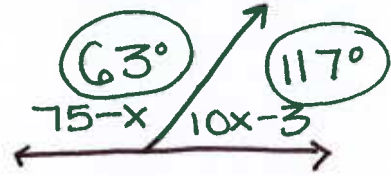
$$7x - 22 = 90$$

$$\begin{array}{r} +22 \quad +22 \\ \hline 7x = 112 \end{array}$$

$$x = 16$$

add to 180°

- 2) Angles 3 and 4 are supplementary. If $m\angle 3 = 75 - x$ and $m\angle 4 = 10x - 3$, find the value of x and the measures of the angles.



$$75 - x + 10x - 3 = 180$$

$$9x + 72 = 180$$

$$\begin{array}{r} -72 \quad -72 \\ \hline 9x = 108 \end{array}$$

$$\frac{9x}{9} = \frac{108}{9}$$

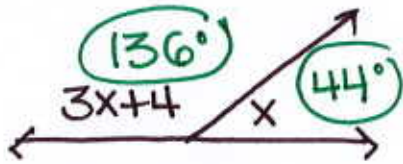
$$x = 12$$

$$75 - 12 = 63^\circ$$

$$10(12) - 3 = 117^\circ$$

- 3) An angle is four more than three times its supplement. Find the measure of both angles.

add to 180°



$$3x + 4 + x = 180$$

$$4x + 4 = 180$$

$$\begin{array}{r} -4 \quad -4 \\ \hline 4x = 176 \end{array}$$

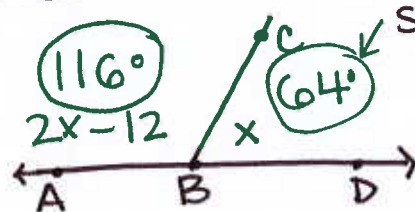
$$\frac{4x}{4} = \frac{176}{4}$$

$$x = 44$$

$$3(44) + 4 = 136$$

- 4) $\angle ABC$ and $\angle CBD$ are a linear pair. The measure of $\angle ABC$ is twelve less than twice the measure of $\angle CBD$. Find the measure of the smaller angle.

makes line (adds to 180°)



smaller angle

$$2x - 12 + x = 180$$

$$3x - 12 = 180$$

$$\begin{array}{r} +12 \quad +12 \\ \hline 3x = 192 \end{array}$$

$$\frac{3x}{3} = \frac{192}{3}$$

$$x = 64$$

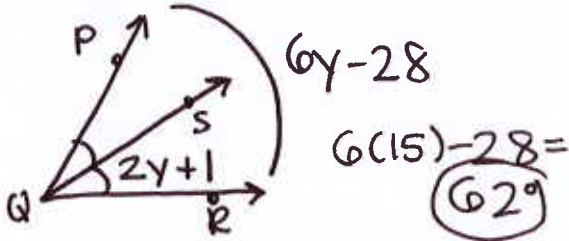
$$2(64) - 12 = 116$$

Problem Solving Strategies

- draw a picture
- look for key words
- re-read (to make sure you solved the problem)

cut in HALF

- 5) \overline{QS} bisects $\angle PQR$. If $\angle PQR = 6y - 28$ and $\angle RQS = 2y + 1$, find $m\angle PQR$.



$$2y+1 + 2y+1 = 6y-28$$

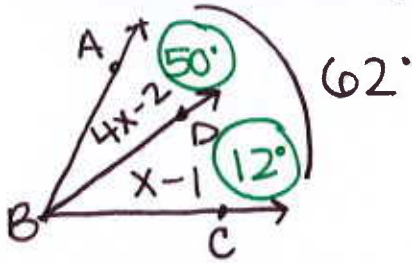
$$4y+2 = 6y-28$$

$$\begin{array}{r} -4y \quad -4y \\ \hline 2 = 2y - 28 \\ +28 \quad +28 \end{array}$$

$$\frac{30}{2} = \frac{2y}{2} \quad \boxed{y=15}$$

next to

- 7) $\angle ABC$ is formed by adjacent angles $\angle ABD$ and $\angle CBD$. If $m\angle ABC = 62$, $m\angle ABD = 4x - 2$, and $m\angle CBD = x - 1$, find the measure of both angles.



$$4x-2 + x-1 = 62$$

$$\begin{array}{r} 5x-3 = 62 \\ +3 \quad +3 \end{array}$$

$$\frac{5x}{5} = \frac{65}{5}$$

$$x = 13$$

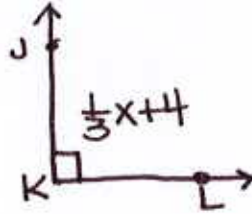
$\angle ABD$
 $4(13)-2 = 50^\circ$

$\angle CBD$
 $13-1 = 12^\circ$

Vertex

$= 90^\circ$

- 6) $\angle JKL$ is a right angle. If $m\angle JKL = \frac{1}{3}x + 4$, find the value of x .



$$\frac{1}{3}x + 4 = 90$$

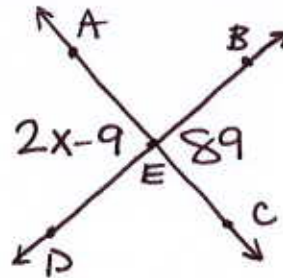
$$\begin{array}{r} \frac{1}{3}x = 86 \\ \frac{1}{3} \quad \frac{1}{3} \end{array}$$

$$\boxed{x = 258}$$

lines

cross

- 8) \overline{AC} and \overline{BD} intersect at E . If $m\angle AED = 2x - 9$ and $m\angle BEC = 89$, find the value of x .



VERTICAL ANGLES

$$\begin{array}{r} 2x-9 = 89 \\ +9 \quad +9 \end{array}$$

$$\frac{2x}{2} = \frac{98}{2}$$

$$\boxed{x = 49}$$

Do bisect and intersect mean the same thing?

No! Bisect means to cut in HALF while intersect means CROSS.

Ratios:

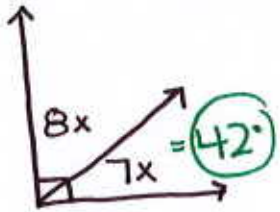
- 1:10
- 2:20 (x2)
- 3:30 (x3)
- x:10x (times x)

Challenge!

add to 90°

9) Two complementary angles are in the ratio of 7:8. What is the number of degrees in the smaller angle?

7:8
↓
7x:8x



$$8x + 7x = 90$$

$$\frac{15x}{15} = \frac{90}{15}$$

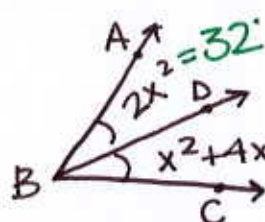
$$x = 6$$

$$7(6) = 42$$

$$8(6) = 48$$

cut in HALF

10) \overrightarrow{BD} bisects $\angle ABC$. If $m\angle ABD = 2x^2$ and $m\angle CBD = x^2 + 4x$, find $m\angle ABC$



$$2x^2 = x^2 + 4x$$

$$-x^2 - x^2$$

$$x^2 = 4x$$

$$-4x - 4x$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x = 0$$

$$x = 4$$

factor GCF

x^2... get all terms on one side!

$\angle ABC$ IS 64°

this ans. doesn't make sense. an angle $\neq 0$

$$2(4)^2 = 32$$

$$(4)^2 + 4(4) = 32$$

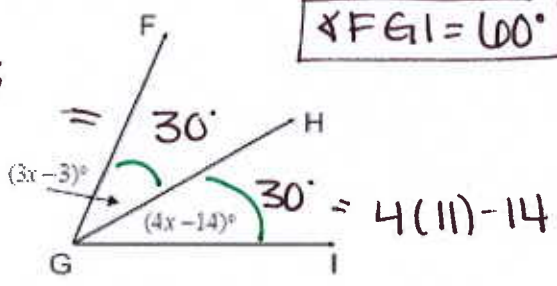
Practice

cut in HALF

1) \overrightarrow{GH} bisects $\angle FGI$. Find $m\angle FGI$.

$\angle FGI = 60^\circ$

$$3(11) - 3 = 30$$



$$3x - 3 = 4x - 14$$

$$-3x \quad -3x$$

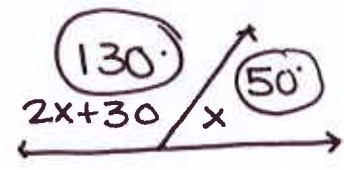
$$-3 = x - 14$$

$$+14 \quad +14$$

$$11 = x$$

add to 180°

2) The measure of the supplement of an angle is 30 degrees more than twice the measure of the angle. Find the degree measure of the angle.



$$2x + 30 + x = 180$$

$$3x + 30 = 180$$

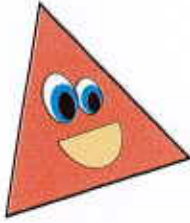
$$-30 \quad -30$$

$$3x = 150$$

$$\frac{3x}{3} = \frac{150}{3}$$

$$x = 50$$

$$2(50) + 30 = 130$$



Triangles

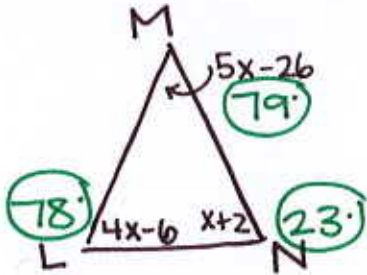
	Definition	Picture
Scalene	a triangle with 3 different sides (no sides are \cong) *this means no angles are \cong	
Isosceles	a triangle with 2 \cong sides *this means that 2 angles (the base angles) are \cong	
Equilateral	a triangle with all sides congruent and all angles congruent!	
Right	a triangle with a 90° angle. *legs can be \cong in an isosceles right Δ	

\cong
congruent
(equal)

★ The sum of the angles of a triangle is always 180° ★

Triangle Word Problems

- 1) In $\triangle LMN$, $m\angle L = 4x - 6$, $m\angle M = 5x - 26$, and $m\angle N = x + 2$. Find the measure of each angle of the triangle. What type of triangle is $\triangle LMN$?



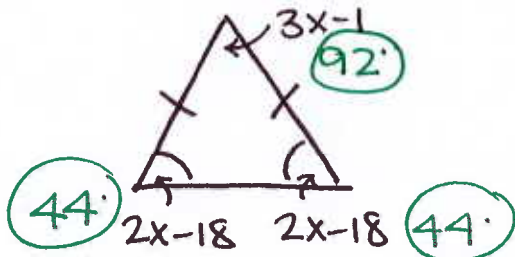
$$\begin{aligned} L: 4(21) - 6 &= 78^\circ \\ M: 5(21) - 26 &= 79^\circ \\ N: 21 + 2 &= 23^\circ \end{aligned}$$

$$5x - 26 + x + 2 + 4x - 6 = 180$$

$$\begin{array}{r} 10x - 30 = 180 \\ +30 \quad +30 \\ \hline 10x = 210 \\ \frac{10x}{10} = \frac{210}{10} \\ x = 21 \end{array}$$

Since no angles are the same, $\triangle LMN$ is **SCALENE!**

- 2) The measure of the base angles of an isosceles triangle are each $2x - 18$. The vertex angle measures $3x - 1$. Find the measure of each angle of the triangle.

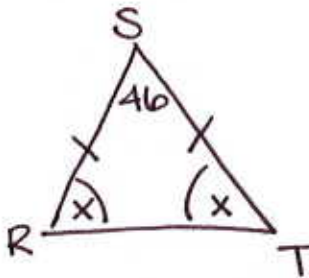


$$\begin{aligned} 3(31) - 1 &= 92^\circ \\ 2(31) - 18 &= 44^\circ \end{aligned}$$

$$3x - 1 + 2x - 18 + 2x - 18 = 180$$

$$\begin{array}{r} 7x - 37 = 180 \\ +37 \quad +37 \\ \hline 7x = 217 \\ \frac{7x}{7} = \frac{217}{7} \\ x = 31 \end{array}$$

- 3) In $\triangle RST$, $m\angle RST = 46^\circ$ and $\overline{RS} \cong \overline{ST}$. Find $m\angle STR$.

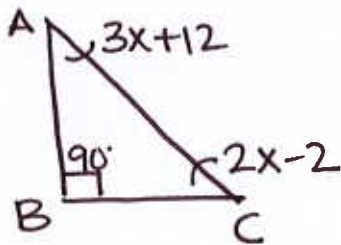


↑
congruent

$$46 + x + x = 180$$

$$\begin{array}{r} 2x + 46 = 180 \\ -46 \quad -46 \\ \hline 2x = 134 \\ \frac{2x}{2} = \frac{134}{2} \\ \boxed{x = 67} \end{array}$$

- 4) In right triangle $\triangle ABC$, $\angle B$ is a right angle. If $m\angle A = 3x + 12$ and $m\angle C = 2x - 2$, find the measure of each angle in $\triangle ABC$.



$$3x + 12 + 2x - 2 + 90 = 180$$

$$5x + 100 = 180$$

$$\begin{array}{r} 5x + 100 = 180 \\ -100 \quad -100 \\ \hline 5x = 80 \end{array}$$

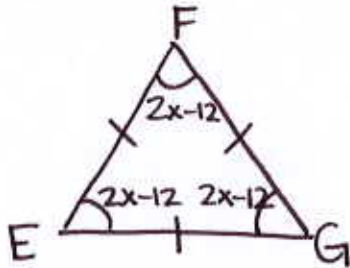
$$x = 16$$

$$\angle A = 3(16) + 12 = \boxed{60^\circ}$$

$$\angle C = 2(16) - 2 = \boxed{30^\circ}$$

- 5) In equilateral triangle $\triangle EFG$, $m\angle E = 2x - 12$. Find the value of x .

all sides \cong



*all angles \cong

$$2x - 12 + 2x - 12 + 2x - 12 = 180$$

$$6x - 36 = 180$$

$$\begin{array}{r} 6x - 36 = 180 \\ +36 \quad +36 \\ \hline 6x = 216 \end{array}$$

$$x = \boxed{36}$$

Side and Angle Relationships

* Shortest side is across from the smallest angle

* Longest side is across from largest angle.

Shortest side \longrightarrow Longest Side

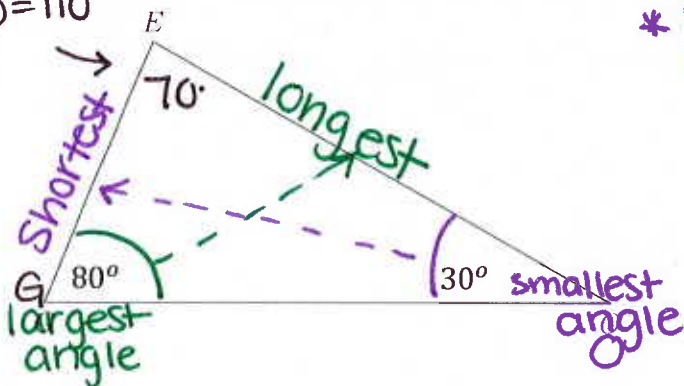
\overline{GE} , \overline{GO} , \overline{EO}

Smallest Angle \longrightarrow Largest Angle

$\angle O$, $\angle E$, $\angle G$

$$80 + 30 = 110^\circ$$

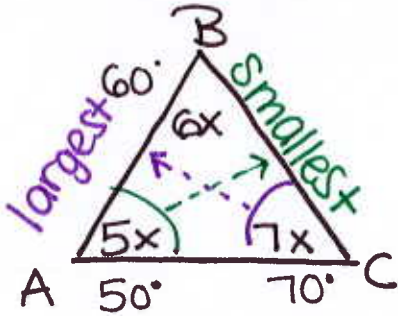
$$\begin{array}{r} 180 \\ -110 \\ \hline 70^\circ \end{array}$$



Examples

$$5x : 6x : 7x$$

- 1) The measures of the angles of $\triangle ABC$ are in the ratio 5:6:7 respectively. List the sides in order from smallest to largest.



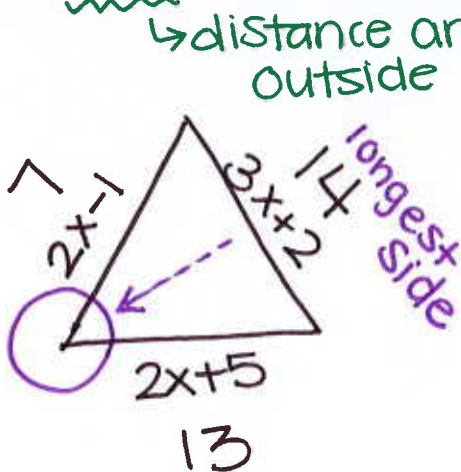
$$5x + 6x + 7x = 180$$

$$\frac{18x}{18} = \frac{180}{18}$$

$$x = 10$$

SMALLEST \rightarrow LARGEST
 \overline{BC} , \overline{AC} , \overline{AB}

- 2) The sides of a triangle can be represented by the expressions $2x - 1$, $3x + 2$, and $2x + 5$. If the perimeter of the triangle is 34 inches, circle the largest angle.

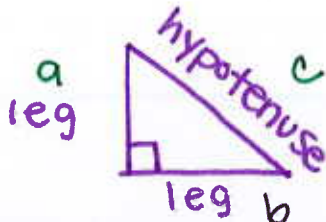


$$2x - 1 + 3x + 2 + 2x + 5 = 34$$

$$7x + 6 = 34$$

$$\frac{7x}{7} = \frac{28}{7}$$

$$x = 4$$

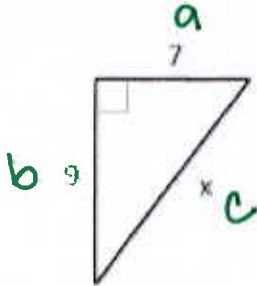


Finding the Third Side of a Right Triangle

$$a^2 + b^2 = c^2$$

PYTHAGOREAN THM

Find the length of the third side of the triangles below to the nearest hundredth.



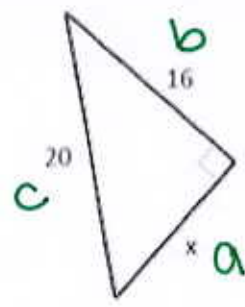
$$a^2 + b^2 = c^2$$

$$7^2 + 9^2 = x^2$$

$$49 + 81 = x^2$$

$$\sqrt{130} = \sqrt{x^2}$$

$$11.40 = x$$



$$a^2 + b^2 = c^2$$

$$x^2 + 16^2 = 20^2$$

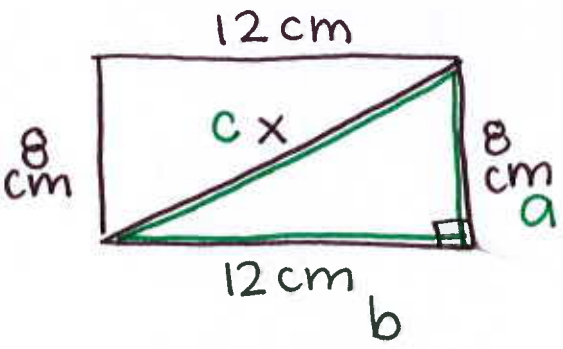
$$x^2 + 256 = 400$$

$$-256 \quad -256$$

$$\sqrt{x^2} = \sqrt{144}$$

$$x = 12$$

Given a rectangle with a width of 8 cm and a length of 12 cm, find the length of the diagonal to the nearest tenth.



$$a^2 + b^2 = c^2$$

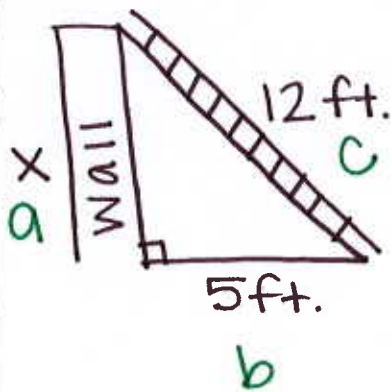
$$12^2 + 8^2 = x^2$$

$$144 + 64 = x^2$$

$$\sqrt{208} = \sqrt{x^2}$$

$$14.4 = x$$

A painter leans a 12-foot ladder up against a wall. If the base of the ladder is 5 feet away from the wall, how high up the wall, to the nearest tenth, does the ladder reach?



$$a^2 + b^2 = c^2$$

$$x^2 + 5^2 = 12^2$$

$$x^2 + 25 = 144$$

$$-25 \quad -25$$

$$\sqrt{x^2} = \sqrt{119}$$

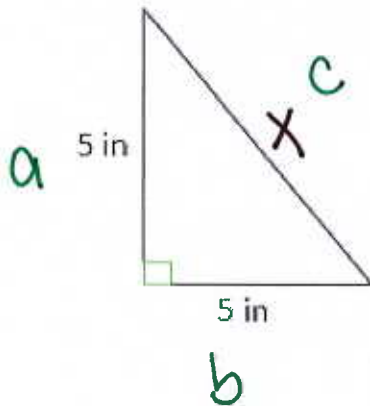
$$x = 10.9 \text{ ft}$$

What is a perfect square?

1, 4, 9, 16, 25, 36, 49, 64,
81, 100, 121, ...

created by multiplying
a # by itself
ex: $6 \cdot 6 = 36$

Find the length of the third side of the triangles below in simplest radical form.



$$a^2 + b^2 = c^2$$

$$5^2 + 5^2 = x^2$$

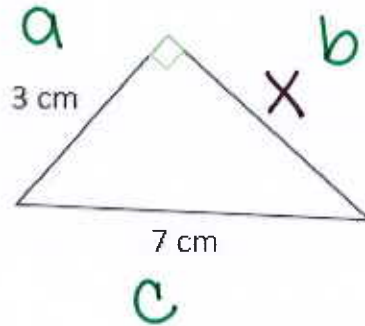
$$25 + 25 = x^2$$

$$\sqrt{50} = \sqrt{x^2}$$

$$\sqrt{25} \cdot 2 = x$$

$$5\sqrt{2} = x$$

$$\begin{array}{r} 50 \\ \textcircled{1} 50 \\ 2, 25 \\ 5, 10 \end{array}$$



$$a^2 + b^2 = c^2$$

$$3^2 + x^2 = 7^2$$

$$9 + x^2 = 49$$

$$\begin{array}{r} -9 \\ \hline \sqrt{x^2} = \sqrt{40} \end{array}$$

$$x = \sqrt{40}$$

$$x = 2\sqrt{10}$$

$$x = 2\sqrt{10}$$

$$\begin{array}{r} 40 \\ \textcircled{1} 40 \\ 2, 20 \\ \textcircled{4} 10 \\ 5, 8 \end{array}$$

What is simplest radical form?

① Find the largest perfect square to go into the # under $\sqrt{\quad}$ and pull it out.

Ex: $\sqrt{32}$

$$\sqrt{\textcircled{16} \cdot 2} = 4\sqrt{2}$$

in calc type
 $y = 32/x$

Factors of 32

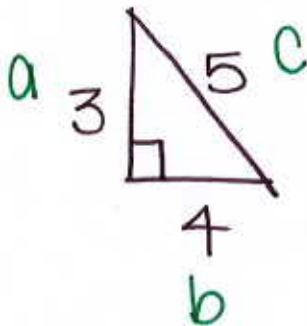
①, 32
2, 16 ← largest perfect square
④, 8

Prove it's a right triangle:

longest side =
hypotenuse
→

A triangle has sides with lengths of 3 mm, 4 mm, and 5 mm. Is this triangle a right triangle? Justify.

* if it's a right triangle, the pythagorean thm must hold true!



$$a^2 + b^2 = c^2$$

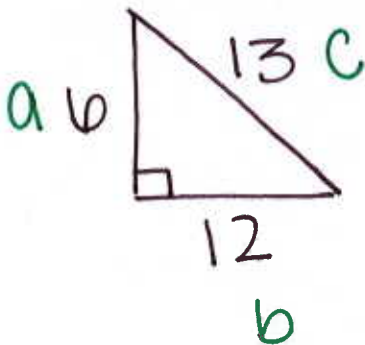
$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

$$25 = 25 \quad \checkmark$$

these sides
make a
right Δ !

A triangle has sides with lengths of 6 cm, 12 cm, and 13 cm. Is this triangle a right triangle? Justify.



$$a^2 + b^2 = c^2$$

$$6^2 + 12^2 = 13^2$$

$$36 + 144 = 169$$

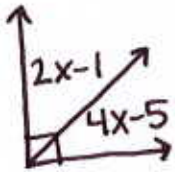
$$180 \neq 169$$

No... these sides do NOT form a right triangle as the pythagorean thm does not hold true.

Mixed Review

add to 90°

- 1) $\angle 1$ and $\angle 2$ are complementary. If $m\angle 1 = 2x - 1$ and $m\angle 2 = 4x - 5$, find $m\angle 2$



$$2x - 1 + 4x - 5 = 90$$

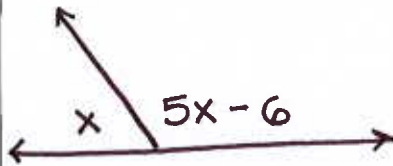
$$\begin{array}{r} 6x - 6 = 90 \\ +6 \quad +6 \\ \hline 6x = 96 \\ \frac{6x}{6} = \frac{96}{6} \end{array}$$

$$x = 16$$

$$m\angle 2 = 4(16) - 5 = \boxed{59^\circ}$$

add to 180°

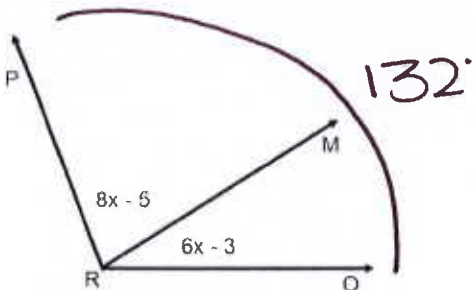
- 2) $\angle 1$ and $\angle 2$ are supplementary. If $m\angle 1$ is six less than five times $m\angle 2$, find $m\angle 1$



$$\begin{array}{r} x + 5x - 6 = 180 \\ 6x - 6 = 180 \\ +6 \quad +6 \\ \hline 6x = 186 \\ \frac{6x}{6} = \frac{186}{6} \\ x = 31 \end{array}$$

$$m\angle 1 = 5(31) - 6 = \boxed{149^\circ}$$

- 3) If $m\angle PRO = 132^\circ$, find $m\angle PRM$



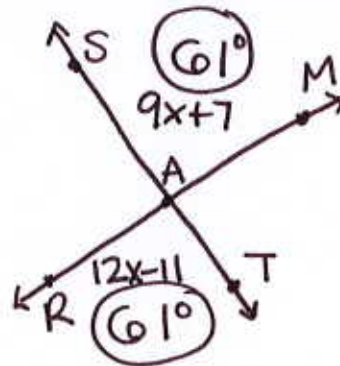
$$8x - 5 + 6x - 3 = 132$$

$$\begin{array}{r} 14x - 8 = 132 \\ +8 \quad +8 \\ \hline 14x = 140 \\ \frac{14x}{14} = \frac{140}{14} \end{array}$$

$$x = 10$$

$$m\angle PRM = 8(10) - 5 = \boxed{75^\circ}$$

- 4) \overline{ST} and \overline{MR} intersect at A. If $m\angle SAM = 9x + 7$ and $m\angle RAT = 12x - 11$, find $m\angle SAM$ and $m\angle MAT$.



VERTICAL ANGLES

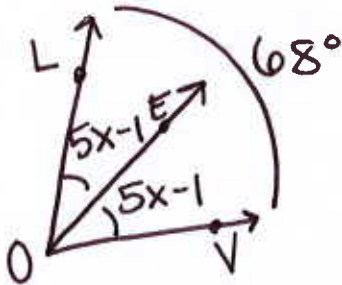
$$\begin{array}{r} 9x + 7 = 12x - 11 \\ -9x \quad -9x \\ \hline 7 = 3x - 11 \\ +11 \quad +11 \\ \hline 18 = 3x \\ \frac{18}{3} = \frac{3x}{3} \\ 6 = x \end{array}$$

$$x = 6$$

$$m\angle SAM = 9(6) + 7 = \boxed{61^\circ}$$

cut in half

5) $\angle LOV$ is bisected by \overline{OE} . If $m\angle LOV = 68^\circ$, and $m\angle LOE = 5x - 1$, find x .

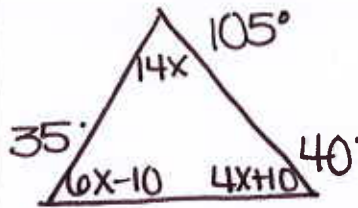


$$5x - 1 + 5x - 1 = 68$$

$$10x - 2 = 68$$

$$\begin{array}{r} +2 \quad +2 \\ \hline 10x = 70 \\ \hline 10 \quad 10 \\ \hline x = 7 \end{array}$$

6) The measures of the angles of a triangle can be represented by the expressions $14x$, $6x - 10$, and $4x + 10$. Find the value of x . What type of triangle is this?



$$14x + 6x - 10 + 4x + 10 = 180$$

$$24x = 180$$

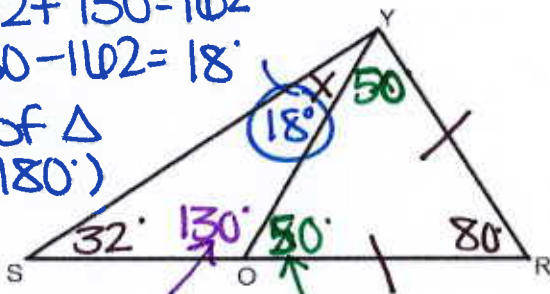
$$\begin{array}{r} 24x = 180 \\ \hline 24 \quad 24 \\ \hline x = 7.5 \end{array}$$

Scalene Δ (you can also call it obtuse)

7) ΔYOR is isosceles with $\overline{YR} \cong \overline{OR}$ and $m\angle YRS = 80^\circ$. If $m\angle YSR = 32^\circ$, find $m\angle SYO$.

$$32 + 130 = 162$$

$$180 - 162 = 18^\circ$$



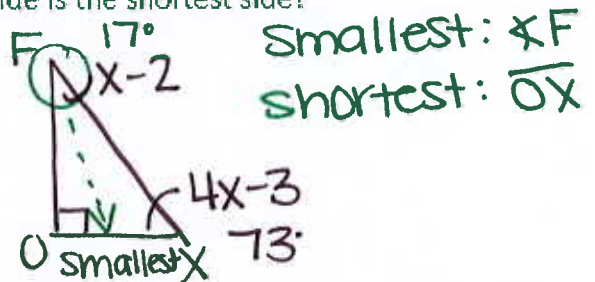
$$\begin{array}{r} 180 \\ - 50 \\ \hline 130 \end{array}$$

(linear pair)

$$\begin{array}{r} 180 \\ - 80 \\ \hline 100 \end{array}$$

$100 \div 2 = 50^\circ$
(base \angle 's of an isosceles Δ are \cong)

8) ΔFOX is a right triangle with a right angle at $\angle O$. If $m\angle F = x - 2$ and $m\angle X = 4x - 3$, find the measure of the smallest angle. What side is the shortest side?



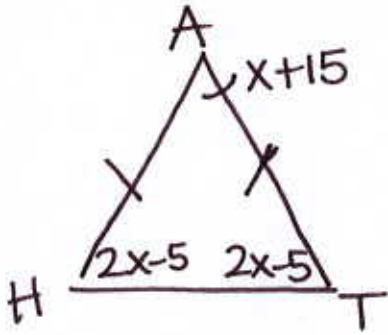
$$x - 2 + 4x - 3 + 90 = 180$$

$$5x + 85 = 180$$

$$\begin{array}{r} -85 \quad -85 \\ \hline 5x = 95 \end{array}$$

$$\begin{array}{r} 5x = 95 \\ \hline 5 \quad 5 \\ \hline x = 19 \end{array}$$

- 9) $\triangle HAT$ is an isosceles triangle with $\overline{HA} \cong \overline{TA}$. The measure of the vertex angle is $x + 15$ and the measure of each of the base angles is $2x - 5$. Find the value of x .



$$x + 15 + 2x - 5 + 2x - 5 = 180$$

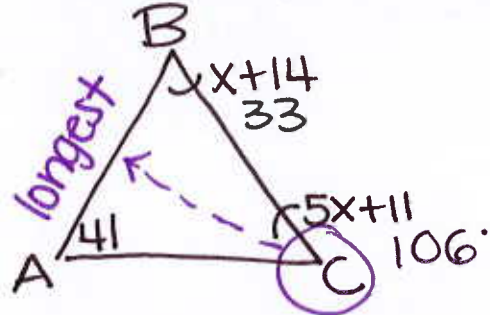
$$5x + 5 = 180$$

$$\begin{array}{r} -5 \quad -5 \\ \hline \end{array}$$

$$\begin{array}{r} 5x = 175 \\ \hline \end{array}$$

$$\boxed{x = 35}$$

- 10) In $\triangle ABC$, $m\angle A = 41^\circ$, $m\angle B = x + 14$, and $m\angle C = 5x + 11$, find the value of x and identify the longest side of $\triangle ABC$.



$$41 + x + 14 + 5x + 11 = 180$$

$$6x + 66 = 180$$

$$\begin{array}{r} -66 \quad -66 \\ \hline \end{array}$$

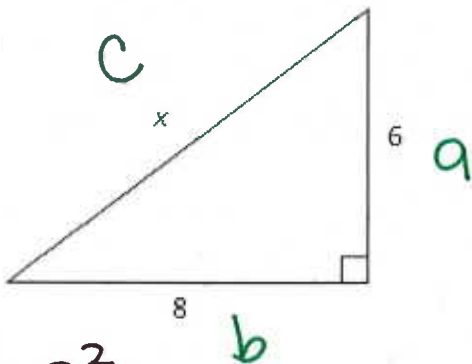
$$6x = 114$$

$$\begin{array}{r} \frac{6}{6} \quad \frac{6}{6} \\ \hline \end{array}$$

$$x = 19$$

longest side = \overline{AB}

- 11) Find the length of the missing side in simplest radical form.



$$a^2 + b^2 = c^2$$

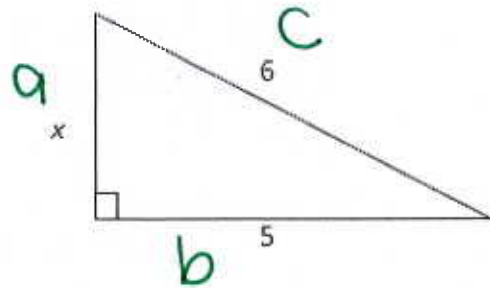
$$6^2 + 8^2 = x^2$$

$$36 + 64 = x^2$$

$$\sqrt{100} = \sqrt{x^2}$$

$$\boxed{10 = x}$$

- 12) Find the length of the missing side in simplest radical form.



$$a^2 + b^2 = c^2$$

$$x^2 + 5^2 = 6^2$$

$$x^2 + 25 = 36$$

$$\begin{array}{r} -25 \quad -25 \\ \hline \end{array}$$

$$\sqrt{x^2} = \sqrt{11}$$

$$\boxed{x = \sqrt{11}}$$