Lesson 1 – Functions

Function Definition and Representations

A **function** is one of the most important concepts in Algebra. Consider the following examples.

Tony and Maria attend different schools that each have a vending machine in the cafeteria.

- Tony's favorite snack, potato chips, are in the location labeled A7. Each time he pays and inputs A7, potato chips come out.
- Maria's favorite snack, chocolate bars, are in the location labeled B4. Each time she pays and inputs B4, the vending machine drops a chocolate bar, but also mixed nuts.

Something is a **function** if every x-value (or input) in the domain is assigned to only one y-value (or output) in the range.

1. Based on this definition, which person's vending machine would be an example of something that is a function? Explain how you know.

   Tony's vending machine matches the definition of a function. Each time he inputs A7; only potato chips come out.

2. Explain why the other person's vending machine is not a function.

   Maria's vending machine is not a function since for one input (B4) there are two outputs (chocolate bar and mixed nuts).

The **domain** of a function is the set of all inputs with outputs. The set of all outputs a function has is its **range**.

3. In the example of the vending machine, what would represent the domain and range?

   Domain: The button combinations for each snack. ***Educator Note:*** Students may suggest money as an input too, which is correct.

   Range: The snacks

A function is usually some mathematical rule that tells you what to do with x in order to get y.

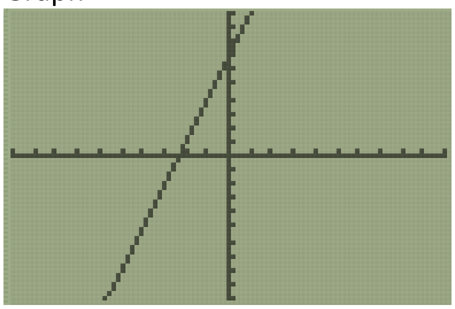
Consider the linear function

$$y = 3x + 7$$

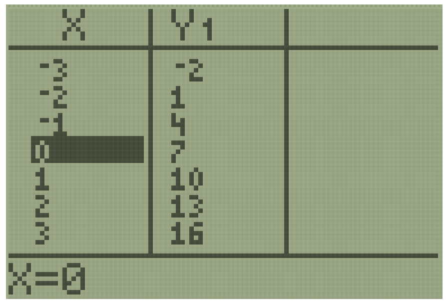
This rule tells you to multiply each x-value by 3 then add 7 to get the y-value.

Here are other ways to represent $y = 3x + 7.$

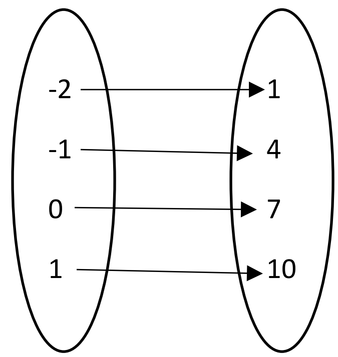
Graph



Table



Arrow Diagram (mapping)



Every x-value has exactly one y-value.

4. Solve the following Regents question.

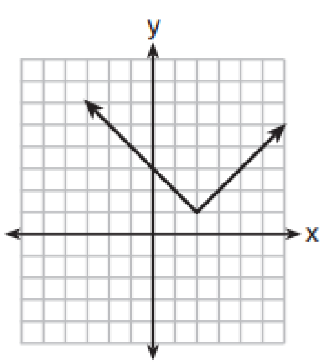
7 Which relation does *not* represent a function?

x	1	2	3	4	5	6
y	3.2	4	5.1	6	7.4	8.8

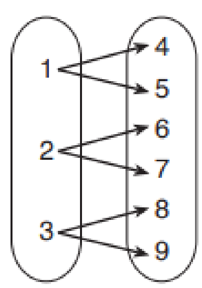
(1)

$y = 3\sqrt{x+1} - 2$

(3)



(2)



(4)

Choice 4 is not a function since each input goes to two different outputs.

An equivalent way to write this linear function is with <u>function notation</u>: $\boldsymbol{f(x) = 3x + 7}$

You say **"f of x"** when you see $\boldsymbol{f(x)}$ and it is another way of writing the variable $\boldsymbol{y}$.

Function notation is used to input numbers for **x**.

For instance, consider $\boldsymbol{f(8)}$**.** This means, "find the y-value when x equals 8." For a simple linear function such as $\boldsymbol{f(x) = 3x + 7}$, you may be able to find y in your head, or with the home screen of your calculator. Here is how to calculate $\boldsymbol{f(8)}$ *algebraically*.

$$\boldsymbol{f(8) = 3 \cdot 8 + 7}$$

$$\boldsymbol{f(8) = 24 + 7}$$

$$\boldsymbol{f(8) = 31}$$

5. Determine the value of $\boldsymbol{f(3)}$**,** $\boldsymbol{f(25)}$, and of $\boldsymbol{f(-6)}$ algebraically.

$$\boldsymbol{f(3) = 3(3) + 7 = 9 + 7 = 16}$$

$$\boldsymbol{f(25) = 3(25) + 7 = 82}$$

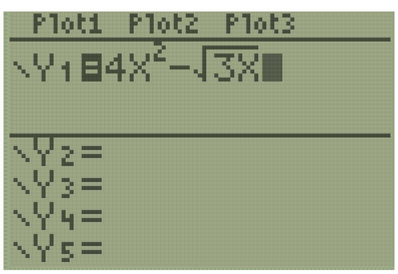
$$\boldsymbol{f(-6) = 3(-6) + 7 = -18 + 7 = -11}$$

Linear functions like $\boldsymbol{f(x) = 3x + 7}$ are the easiest ones to work with.

<u>Using the TABLE</u>

Consider a non-linear function $\boldsymbol{g(x) = 4x^2 - \sqrt{3x}}$**.** Notice that this function has been named $g(x)$ "g of x". We can use any letter to name a function, which is helpful when a problem involves more than one function. Let's explore $g(x)$ with the calculator.

Every function can be represented with a <u>table</u> or a <u>graph</u>.



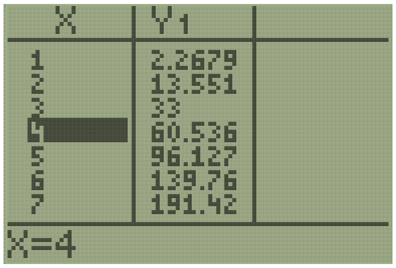
All function rules can be put into **Y=**

The 2 on top of the x is an <u>exponent</u>. To get an exponent, **press the ^ key** and then the value of the exponent. Since 2 is a very common exponent, you can also press **the** $\boldsymbol{x^2}$ **key**. Notice that anything you type will stay in the exponent until you press the right arrow button →.

Notice that this function involves the <u>square root</u> of **3x**. This symbol $\sqrt{}$ is called a <u>radical</u>. Find the radical symbol on your calculator by pressing **2nd** and $\boldsymbol{x^2}$.

From the **Y=** menu, we can find specific values of $g(x)$ by using tables or graphs.

Alejandro wants to find $\boldsymbol{g(4)}$ using a table.

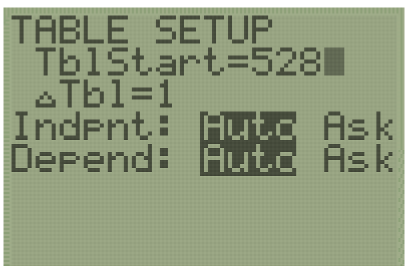


Press **2nd, GRAPH**

Use the up and down arrows until you get to x = 4.

$$g(4) = 60.536$$

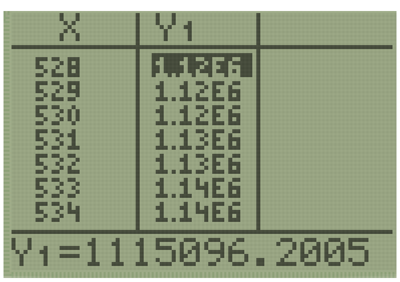
Next, Alejandro wants to find $\boldsymbol{g(528)}$ using a table. He definitely doesn't want to press the down arrow that many times.



He presses **2nd, WINDOW** and changes **TblStart** to 528

$\boldsymbol{\Delta Tbl = 1}$ means that the x-values will count by 1.

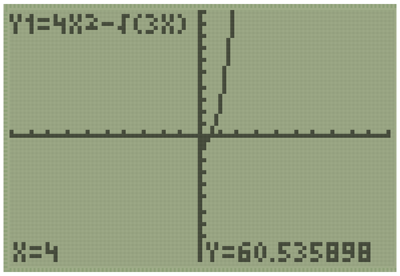
Don't worry about the rest.



When Alejandro presses **2nd GRAPH**, his table starts with **x = 528**. The y-value is too large to display properly unless he highlights it using the arrow keys. This sometimes happens when x is very large.

Alejandro concludes that $g(528) = 1115096.2005$

Using the GRAPH



Tatiana is working with the same function, $\boldsymbol{g(x) = 4x^2 - \sqrt{3x}}$

Tatiana wants to find $\boldsymbol{g(4)}$ using a graph.

Press **GRAPH**. If your viewing window is different, press **ZOOM, 6** to get a standard window.

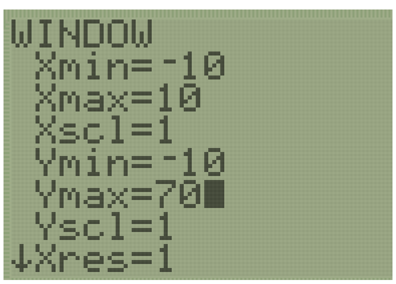
Press **TRACE**, **4**, **ENTER**

Tatiana notices that when x = 4, y = 60.535898.

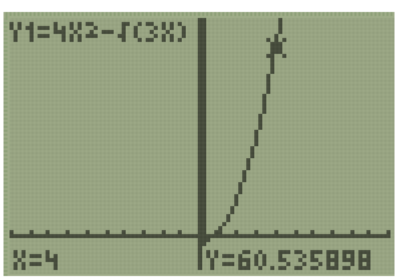
Alejandro's and Tatiana's answers for $\boldsymbol{g(4)}$ are slightly different, but that's okay.

Tatiana notices that her calculator tells her that $g(4) = 60.535898$ at the bottom of her screen, but that she can't see the y-value 60.535898 on the graph of the function. She counts along the y-axis and notices that it only goes up to 10.

She decides to make her graph show more y-values.



She presses **WINDOW** and sets her **maximum y-value** to 70.



When she presses **GRAPH**, the y-axis goes up to 70.

When she presses **TRACE**, **4**, **ENTER**, she can see the location of $g(4)$ on the graph.

In general, make the **Xmax** and **Ymax** higher than the number you want, and make the **Xmin** and **Ymin** lower than the number you want. You never need to change the **Xscl** or **Yscl**, but if you do, they tell the calculator how far to space the marks on the x-axis and y-axis.

Try the following Regents questions. Try to use the graphing calculator tables and graphs, and what you know about functions, to answer them.

6.

The function $g(x)$ is defined as $g(x) = -2x^2 + 3x$. The value of $g(-3)$ is

(1) -27 (3) 27

(2) -9 (4) 45

Choice 1

7.

If $k(x) = 2x^2 - 3\sqrt{x}$, then $k(9)$ is

(1) 315 (3) 159

(2) 307 (4) 153

Choice 4

8.

Marc bought a new laptop for \$1250. He kept track of the value of the laptop over the next three years, as shown in the table below.

Years After Purchase	Value in Dollars
1	1000
2	800
3	640

Which function can be used to determine the value of the laptop for x years after the purchase?

(1) $f(x) = 1000(1.2)^x$

(2) $f(x) = 1000(0.8)^x$

(3) $f(x) = 1250(1.2)^x$

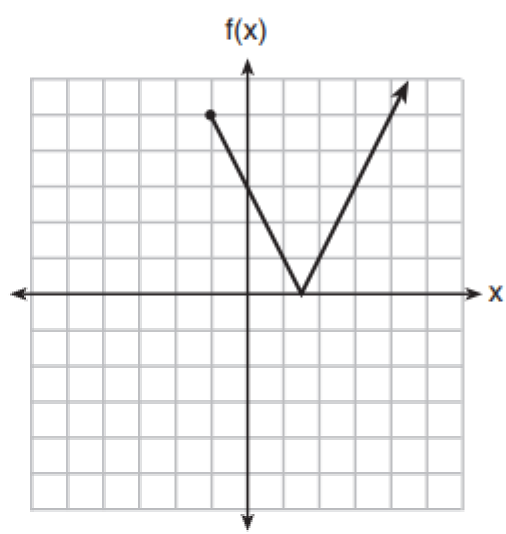
(4) $f(x) = 1250(0.8)^x$

Choice 4

Educator Note: Try each choice in Y= and choose the answer with the same table

9.

The function $f(x)$ is graphed below.



The domain of this function is

(1) all positive real numbers

(2) all positive integers

(3) $x \geq 0$

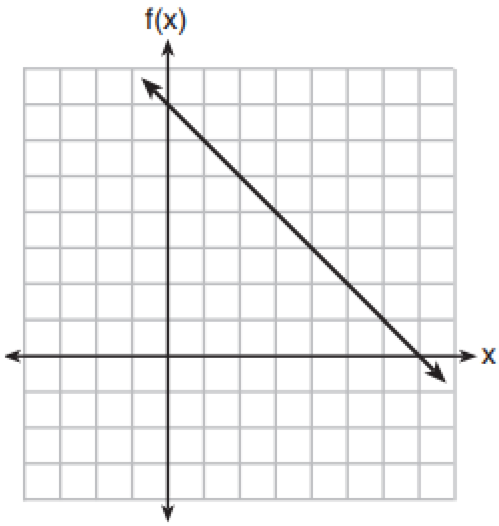
(4) $x \geq -1$

Choice 4

Educator Note: Notice that there are no y-values for x-values below -1.

10.

The functions $f(x)$, $q(x)$, and $p(x)$ are shown below.



$$q(x) = (x - 1)^2 - 6$$

x	p(x)
2	5
3	4
4	3
5	4
6	5

When the input is 4, which functions have the same output value?

(1) $f(x)$ and $q(x)$, only

(2) $f(x)$ and $p(x)$, only

(3) $q(x)$ and $p(x)$, only

(4) $f(x)$, $q(x)$, and $p(x)$

Choice 4

11.

Materials A and B decay over time. The function for the amount of material A is $A(t) = 1000(0.5)^{2t}$ and for the amount of material B is $B(t) = 1000(0.25)^t$, where t represents time in days. On which day will the amounts of material be equal?

(1) initial day, only

(2) day 2, only

(3) day 5, only

(4) every day

Choice 4