



NEW YORK STATE MIGRANT EDUCATION PROGRAM

Title: Fractions to Decimal Fractions: Understanding Equivalence in the Transition using Cuisenaire Rods

Description: Students will understand how fractions and their decimal equivalents represent the same amount. Pre-teaching with equivalence of fractions shows the relationship between the part to the whole. Modeling with Cuisenaire Rods allows students to develop intrinsic ability through conceptual understanding. An extension activity with beginning addition and subtraction of decimals is also included. The outcome will be a deeper understanding of the relationship of fractions to decimals and the relationship of the part to the whole.

Developer: Suzanne K. Fox, Staff Development Specialist, Oswego Center for Instruction, Technology & Innovation (CiTi)

Series: This is the fourth and final module in a series of Professional Development videos about Understanding Fractions through Concrete and Visual designs. This module can be used on its own, but it is recommended to participate in *Visualizing Fractions: Modeling addition and subtraction using Cuisenaire rods* * **Part A like denominators** and ***Part B unlike denominators** prior to this video, for the introduction to Cuisenaire rods and practice using them to model the addition and subtraction of fractions.

Facilitator Guide

INDIVIDUAL ACCESS/SELF-SERVE (*for individuals viewing this module independently*): While a robust conversation between colleagues is an enriching way to learn, so is self-reflection. Read and use this Guide as the Facilitator of your own learning. To get the most out of the activities and questions, make sure you have the recommended handouts and supplies listed, before beginning.

CTLE CREDIT

Group Workshop: If you are facilitating this workshop for your METS, you will have to decide which process you will use for granting CTLE credit. You can use your local LEA process, or the M-TASC process:

- a. Contact M-TASC in advance of the workshop to confirm date and module.
- b. Use the M-TASC Participant Sign-In Sheet, and submit.

- c. Submit Workshop Evaluations via link or hard copy. If you use the Evaluation link, M-TASC will forward the compiled evaluations once you have informed the office that all evaluations are complete.

Individual Access/Self-Serve: For those who would like to request Continuing Teacher Leader Education (CTLE) credit for On-Demand professional development, please complete the CTLE Credit Request for each module. Find the link for this process on the NYS-MEP website:

<https://www.nysmigrant.org/resources/pd>

OBJECTIVES/LEARNING TARGET(S)

- I can use visual models to show the relationship of fractions to the whole.
- I can use visual models to show equivalence and comparison in size and value of fractions using concrete and symbolic modeling.
- I can use visual models to show equivalence between the decimal and fraction form for a value using Cuisenaire rods.
- I can use modeling to add and subtract decimal fractions in the tenths using Cuisenaire rods.

WORKSHOP/MODULE DESIGN

Today's web learning session will allow you to bring fractions and decimal fractions to a concrete modeling version. By engaging your students in these modeling activities, they will understand the relationship of fractions to each other, the relationship of fractions to the whole, as well as understand the relationship of fractions to decimal fractions and begin to work with addition and subtraction of these decimal fractions. All of this will be accomplished using Cuisenaire rods.

CONNECTION TO THE NYS MEP THEORY OF ACTION

- New York State Migrant Education Program Theory of Action
 - **Subject Content and Instruction**
Subject: Focus on assuring that in-school students the foundational skills and strategies to succeed in the classroom and on state and other assessments.
 - **Advocacy to Self-Advocacy:**
Learner independence integrates key (meta) cognitive strategies and subject content knowledge with a focus on creating thinkers; problem solvers; and self-regulated, life-long learners.

SUPPLIES AND MATERIALS

- Video: ***Fractions to Video Fractions***
 - This video is for NYS MEP use only.
 - Use the video link on the NYS migrant website in the Professional Development section for this workshop, "*Fractions to Decimal Fractions: Understanding Equivalence in Making the Transition using Cuisenaire Rods*".

- Participant Handouts
 1. Power Point Notes
 2. Paper Cuisenaire Rods with Units Marked. (These will need to be color copied and cut out prior to the workshop.)
 3. Graph paper – $\frac{1}{4}$ inch (optional)
- 4. Website Resource – *mathies* Learning Tools (email ahead of time so participants can use the link)
 - Optional supplies
 - Device for internet use
 - M-TASC Sign-in Sheet
 - M-TASC Exit Survey/Evaluation

PREPARATION AHEAD OF TIME

The paper Cuisenaire Rods need to be color-printed on cardstock and cut apart so they are ready to use. You can add five minutes to the presentation time and ask participants to cut their own rods apart.

Getting Started

- Disseminate
 - Power Point Notes
 - Paper Cuisenaire Rods with Units Marked (cut apart)
- Start the Video Presentation: *Fractions to Video Fractions* (33 minutes)
Plan for an additional 10 to 15 minutes for discussion.

INTRODUCTION (2 minutes)

- Connection to the Theory of Action
- Learning Targets

REVIEW USING CUISENAIRE RODS TO REPRESENT FRACTIONS (7 minutes)

Activity 1 – Participants are given time to “play” with the rods.

Key Point

- Explore relationships between rods of different lengths.

Facilitator Note: Music plays to denote time for participants to engage in independent activities. When the music stops, the presenter will demonstrate or discuss the solutions on the video. If participants need more time, feel free to pause the video for them to complete the activities.

Activity 2 – Using Cuisenaire rods to model equivalent fractions

1. Can you show the relationship of $\frac{1}{2}$ to 1 whole?
2. Can you show the relationship of $\frac{1}{3}$ to 1 whole?

Key Point

- Model at the concrete level the relationship of the part to the whole.

Activity 3 – Using Cuisenaire rods to model a more complex sense of equivalence

- Use the rods diagramed on the video to figure out different combinations of adding $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ to equal 1 whole.

Key Point

- Move from the pictorial/concrete to the symbolic/abstract in writing the fractional equivalents of one whole.

Activity 4 – Increase understanding of equivalence using addition of fractions**TRANSITION TO DECIMALS****Key Points**

- Cuisenaire Rods
 - Use the Orange Rod to represent 1 whole.
 - Use the White units to represent $\frac{1}{10}$ (0.1).
 - You can use the White units to represent $\frac{10}{100}$ (0.10).
 - The other colors represent decimals in between 0.1 and 1 whole.
- Students start decimal work with tenths and hundredths.
- If you are working the fraction one/tenth, then write the decimal as 0.1.
- If you are working with fraction 10/hundredths, then write the decimal as 0.10.

Activity 5 – Developing a framework for teaching students the transition from fractions to decimals with equivalence (short example)

- Think about how you would describe the changes on the video.

Activity 6 – Developing a framework for teaching students the transition from fractions to decimals with equivalence (longer example)

- Develop talking points to understand the flow of movement from fractions to decimals representing the same amount as a turnkey for teaching.
- The three consecutive slides on the Power Point handout show the changes so participants can watch the video instead of taking notes.
- The video will have a “pause” screen so you can stop the video for participants can use the three slides and practice describing the steps.

Activity 7 (online) *Mathies* Learning Tools – Practice using virtual rods to add and subtract decimals

- Pause video for participants
 - to follow the link to participate online, or
 - to be ready to use the paper rods to participate
- Participants will model these problems along with the video:
 $0.6 + 0.9 + 0.5$
 $0.8 - 0.3$
 $1.6 + 0.5$
 $1.7 - 0.9$

Facilitator Note: To practice modeling addition and subtraction with the presenter, encourage participants to model the addition and subtraction along with the presenter. If internet and computers are available for participants to try the online activity, email the Web Resource-*mathies* Learning Tools handout ahead of time so participants can use the link. Otherwise, participants can model the addition and subtraction with the paper Cuisenaire Rods or use graph paper to draw the rods.

<https://oame.on.ca/CLIPS/swfPlayer.html?swfURL=tools/RelationalRods1.swf&title=Relational%20Rods+>

After using the link, click on the “Decimal Rods” icon to enter, then use the web resource handout for navigation information.

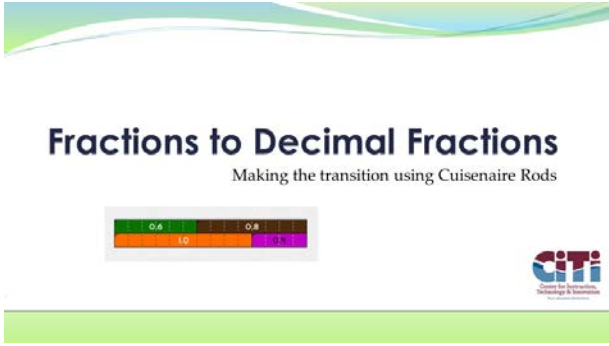
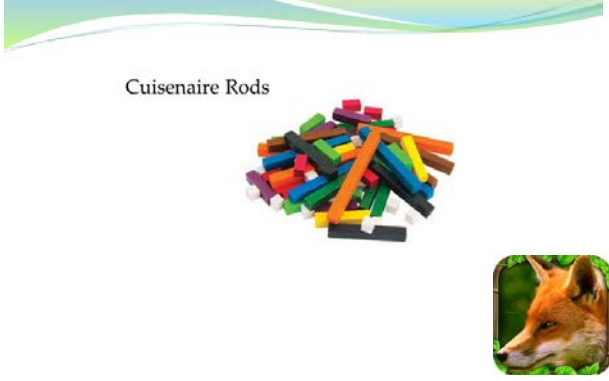

Closure for Group Workshops

- Facilitators are welcome to use the Workshop Evaluation provided on the website or you can use your own version.
- *Continuing Teacher Leader Education (CTLE)* – Follow the CTLE process at your METS program center for staff who are tracking credit.

Closure for Individual Access/Self-Serve

- Complete the Workshop Evaluation and give it to your Director.
- *Continuing Teacher Leader Education (CTLE)* – If you would like to request credit for this module, please follow the CTLE Credit Request process. Find the link for this process on the NYS-MEP website: <https://www.nysmigrant.org/resources/pd>

Facilitator Note: The following Appendix contains the workshop “Talking Points” used by Developer, Suzanne K. Fox, to support your facilitation when participants need something repeated.

	<p>Today’s web-learning session will allow you to bring fractions and decimal fractions to a concrete modeling version. By engaging your students in these modeling activities, they will understand the relationship of fractions to each other, the relationship of fractions to the whole, as well as understand the relationship of fractions to decimal fractions and begin to work with addition and subtraction of these decimal fractions. All of this will be accomplished using Cuisenaire rods. To help you in this understanding, you will need your cut outs of the Cuisenaire rods, 1 sheet of quarter inch graph paper and a device to use an internet resource.</p>
<p>Cuisenaire Rods</p> 	<p>Cuisenaire rods were invented by Georges Cuisenaire, as a primary teacher in Belgium. They became a component of improvement in mathematics education in the 1950s. Now with the realization that students need to “see” the math through the concrete, pictorial, and finally abstract, these simple colored rods make understanding fractions and decimal fractions visible to students.</p>
<p>Always begin with...</p> 	<p>We always begin by playing with the Cuisenaire rods. It gives students time to develop their own “I notice and I wonder” about the rods and what they could be used for. Take a moment to play and find some relationships between the rods. MUSIC Did you notice that the picture that looks like flowers is really representational of a tree diagram in math? Let’s now connect our understanding of fractions and Cuisenaire rods.</p>

Can you show the relationship of $\frac{1}{2}$ to 1 whole?

Can you show the relationship of $\frac{1}{2}$ to 1 whole?

It is so very important for children to develop number sense. Number sense in this slide refers to understanding the relationship between quantities – like part to whole.

Using your rods, see how many combinations you can build using the rods to show the relationship of one half to one whole.

Music

Here are three possible combinations you might have. **CLICK** Notice that even though the rods were not given an assigned number, we were able to show this relationship.

Now watch what happens as these relationships are even further explored. **CLICK** By building on the simple relationship between one half to one whole, we are now able to expand students' understanding to one fourth, and even one eighth to each part of the whole.

Now try modeling the relationship between one third to one whole with your rods.

MUSIC

Here are two models you may have. **CLICK** By using additional rods **CLICK** we can open up and deepen students' understanding of this relationship of one sixth in relation to one third and one whole.

The relationship of fractions to the whole and the concept of equivalence.

"One yellow plus one yellow equals two yellow"

$\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

How many halves (2 yellow one-half rods)

$\frac{1}{2} + \frac{1}{2} = 1$

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$ = 1 (one whole)

Let's look closer at our relationships between one half and the whole. When you introduce this to a child, it may be helpful if you actually say **CLICK** one yellow plus one yellow is the same as or equal to two yellows. Then go on to show how to write this **CLICK** using fraction notation.

Our more advanced model shows equivalence in many ways. If we turn this

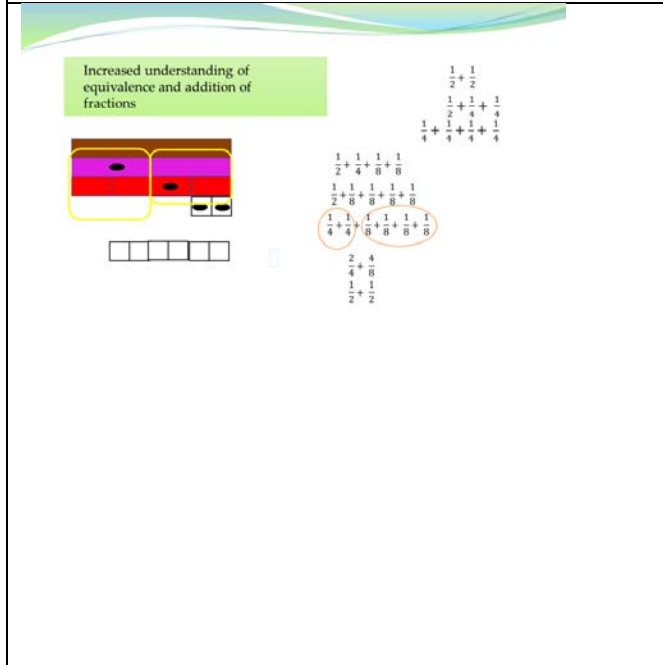
model on its head **CLICK** we can start with the whole and then model how the pieces fit together with the added benefit of doing this both in a concrete and abstract (or mathematical) way.

First, we show **CLICK** one half plus one half. Then we can show one half (the purple rod) **CLICK** plus one fourth plus one fourth (the red rods) also can represent the whole.

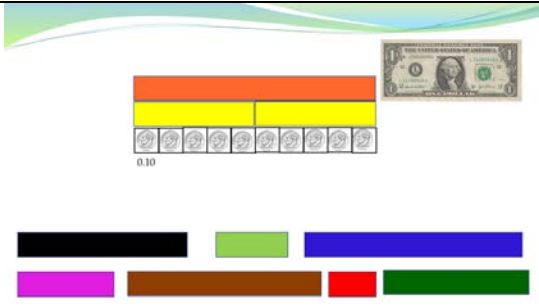

If your student is not sure the red is one fourth, **CLICK** by bringing in two red rods, we can show that four reds make one whole meaning each red is one fourth of the whole. You make think these relationships are pretty obvious, to your student...to show that all of these **CLICK** are the same as **CLICK** one whole is mind blowing!

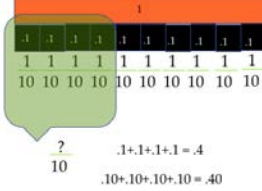
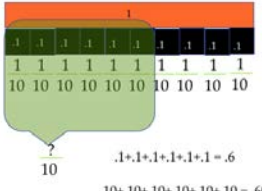
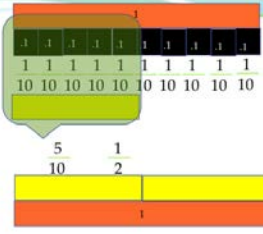
With this in mind, try now to make some combinations using one eighth (that is the white rod) along with our existing rods representing one fourth and one half to show equivalence while still equaling one whole?

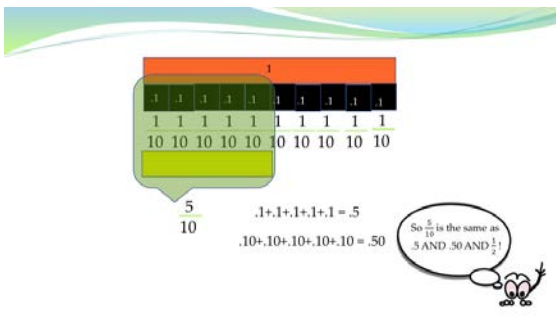
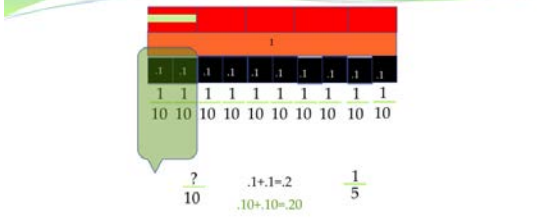
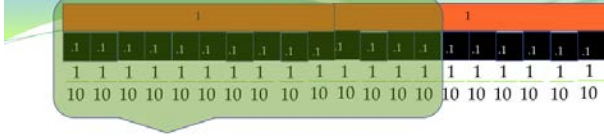
MUSIC

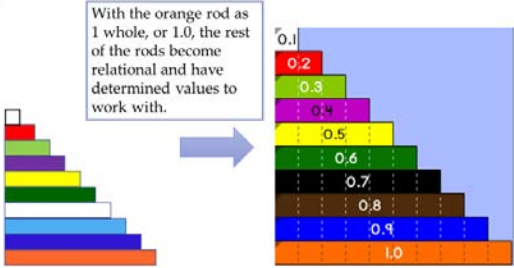
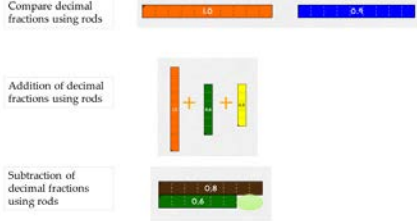
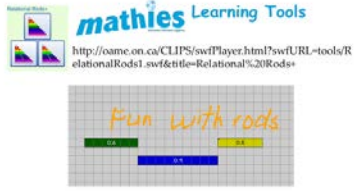
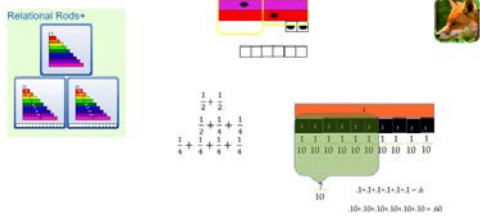


Remembering our sequence using halves and fourths, it may be helpful to add in a whole row of eights. **CLICK** Never underestimate the power of the visual here. Now students can see how one half **CLICK** one fourth and two eighths can be equivalent to one whole. **CLICK** Expanding on this can allow students to come up with their own math sentences **CLICK** as perhaps you have too. As students explore and conceptually understand these relationships, it is always good to keep circling **CLICK** back to the beginning , our part to whole relationship of **CLICK** one half plus one half equals one whole. Take a moment and share with your shoulder partner some other relationships

	<p>you might bring in with a lesson such as this. MUSIC</p>
	<p>Understanding equivalence within the set of fractions is important to establish for when students move to their first encounter with decimal fractions. This starts with using denominators of ten and one hundred to represent tenths and hundredths. If you would get your rods set to model with me as we go through the next set of slides, that would be great. Pause Start out with our orange rod representing one whole. Then we once again use our CLICK two yellow rods and also put in white rods. Notice that ten white rods fit perfectly to make one whole. For now, remove the yellow CLICK so it is just the tenths and one whole. At this point, it may be helpful to use money CLICK and bring in CLICK some dimes to make that real life connection. It is easy to see that tenths are less than one whole, and now students are given the knowledge of how to write a decimal fraction CLICK using tenths. (As a side note, some students may also need the aid of a place value chart.) It is very important that students see that ten tenths is one whole.</p>
	<p>When students are learning how to say the decimal equivalent, always have them refer back to the fraction. Even though it is written differently, it is still the same value and has the same name!</p>

 <p> $\frac{4}{10}$ $.1+.1+.1+.1 = .4$ $.10+.10+.10+.10 = .40$ </p>	<p>Just as we explored equivalence with fractions, we can now explore this relationship between decimals and decimals, and fractions and decimals. By students seeing the fraction and decimal forms at the same time, it is easier for them to make the connection that they represent the same amount. How many tenths are being represented here? We can count one, two, three, four tenths and then represent this CLICK in decimal notation.</p>
 <p> $\frac{6}{10}$ $.1+.1+.1+.1+.1+.1 = .6$ $.10+.10+.10+.10+.10+.10 = .60$ </p>	<p>Let's try another example. We want to find the equivalent decimal form of the fraction. We always start out with the relationship of the part to the whole. CLICK By removing a given number of rods, CLICK we can give students the ability to visualize CLICK this equivalence of six tenths in both forms and also bring in beginnings of addition of decimals. As an added bonus you can always bring back in the CLICK dimes to make that real life connection.</p>
 <p> $\frac{5}{10}$ $\frac{1}{2}$ </p>	<p>This conceptual understanding of the relationship between fractions and decimals using equivalence is the why behind our mathematical conversions. Just doing the steps because the teacher says this will give the right answer does a disservice to the child's sense of curiosity. In this model we further show how five tenths in fraction form is not only equivalent to five tenths in decimal form, it is also equivalent to one half. See if you can follow the visual movements to arrive at this conclusion. CLICK (wait three) CLICK (wait three) CLICK (wait three) CLICK (wait three) CLICK (wait three) Sometimes the visual speaks as loud as the words.</p>

 <p>$\frac{5}{10}$ $.1+.1+.1+.1+.1 = .5$ $.10+.10+.10+.10+.10 = .50$</p> <p>So $\frac{5}{10}$ is the same as .5 AND .50 AND $\frac{1}{2}$!</p>	<p>The visual you just watched was simple and so powerful. Your students now know the WHY behind the equivalence of five tenths representing the same amount as the decimal form of five tenths and even one half!</p>
 <p>$\frac{?}{10}$ $.1+.1 = .2$ $\frac{1}{5}$ $.10+.10 = .20$</p>	<p>Let's practice – Pretend that you are going to narrate this slide. There are 6 add ins to show the relationship between a fraction, its decimal equivalent, and its additional fractional equivalent. After each add in is shown, jot down what you would say happened in the process. If you need more time, just pause and then play. Then have some sharing out with your group about what you see and say.</p> <p>Music – click – music – chick – music – click – music – click – music – click – music – click – music – click – music (to fade out).</p>
 <p>$\frac{?}{10} \rightarrow \frac{14}{10}$</p> <p>$\frac{10}{10} + \frac{4}{10} = 1 \frac{4}{10}$</p>	<p>The next step in your students understanding is moving into a mixed number format.</p> <p>Notice that we always start with the relationship of the part to the whole. By this time students will be able to easily come up with the fraction fourteen tenths. Now we need to look at the whole and what is more than the whole CLICK. Here is the addition sentence in fraction form. CLICK When moving into the decimal form CLICK it is crucial that students understand that the decimal point is after the one. A common mistake when first learning this is that students will count to ten, write ten, and then put in a decimal after the zero. By using the rods, students can see that this sum is exactly one with zero tenths. Looking at the second part, it</p>

	<p>is easy to add up the decimal form CLICK. Now the equivalence is shown again with the CLICK whole and the part of the whole in both formats with the final answer CLICK in decimal format.</p>
	<p>After students understand equivalence between writing a value in decimal and fraction form, Cuisenaire rods can be used to show addition and subtraction of decimals in tenths. A very common mistake would be to misplace the decimal point. The rods help students to see why and where the decimal point should be in relation of the whole to the part.</p>
	<p>I am now going to move to a website that allows virtual manipulation of rods in addition and subtraction. If you have a device with you and would like to model along with me, please pause this video and log onto the site on the next slide.</p>
	<p>Show this slide and then move to the site to model the following: $0.6+0.9+0.5$ $0.8-0.3$ $1.6+0.5$ $1.7-0.9$</p>
	<p>Once again, this teaching model of concrete, pictorial, and abstract or symbolic is a means for students to gain the why numbers behave in relation to each other. This brings us to true mathematical literacy and brings out the mathematical in all of us. Thank you for allowing me to be a part of your math journey.</p>