NEW YORK STATE
MIGRANT EDUCATION PROGRAM

Title: Expressions and Equations: Physical and Mathematical Modeling
Part 2 - Algebraic Expressions

Description: Students will understand how variables behave in the same relationships with each other, as do numbers without using traditional *rules*. Following a brief review of *Expressions and Equations Part 1 – Operations with Integers*, participants will model algebraic expressions using algebra tiles and T-charts. The outcome will be a deeper understanding of algebraic thinking and will assist students as they transition from working with numbers to the language of algebra.

Developer: Suzanne K. Fox, Staff Development Specialist, Oswego Center for Instruction, Technology & Innovation (CiTi)

Series: This is the second in a two-part series on Expressions and Equations using Algebra Tiles. It is recommended to watch these modules in sequence.
Part 1 – Operations with Integers
Part 2 – Algebraic Expressions

Facilitator Guide

INDIVIDUAL ACCESS/SELF-SERVE *(for Individuals viewing this module independently):* While a robust conversation between colleagues is an enriching way to learn, so is self-reflection. Read and use this Guide as the Facilitator of your own learning. To get the most out of the activities and questions, make sure you have the recommended handouts and supplies listed below, before beginning.

CTLE CREDIT

Group Workshop: If you are facilitating this workshop for your METS, you will have to decide which process you will use for granting CTLE credit. You can use your local LEA process, or the M-TASC process:

a. Contact M-TASC in advance of the workshop to confirm date and module.
b. Use the M-TASC Participant Sign-In Sheet and submit.
c. Submit Workshop Evaluations via link or hard copy. If you use the Evaluation link, M-TASC will forward the compiled evaluations once you have informed the office that all evaluations are complete.

Individual Access/Self-Serve: For those who would like to request Continuing Teacher Leader Education (CTLE) credit for On-Demand professional development, please complete the CTLE Credit Request for each module. Find the link for this process on the NYS-MEP website: https://www.nysmigrant.org/resources/pd

OBJECTIVES/LEARNING TARGET(S)

- I can model expressions using physical and mathematical models
  - Identifying essential variables
  - Describing the relationships between the variables (and numbers)
  - Performing the operations

WORKSHOP/MODULE DESIGN

This web learning session will allow you to model algebraic expressions using algebra tiles and t charts. It will enable you and your students to understand how variables behave in the same relationships with each other, as do numbers without using traditional "rules". This ability to have a deeper understanding of algebraic thinking will assist students as they transition from working with the concrete to the abstract in mathematics.

The target audience for this workshop are educators working in grades 6-9 where the emphasis is on conceptual and procedural understanding of making equivalent and simplifying algebraic expressions. This is the second in a two-part series on using the Concrete, Representational, Abstract (CRA) approach to understanding operations with integers and variables.

CONNECTION TO THE NYS MEP THEORY OF ACTION

- New York State Migrant Education Program Theory of Action
  - **Subject Content and Instruction Subject:** Focus on assuring that in-school students the foundational skills and strategies to succeed in the classroom and on state and other assessments.
  - **Advocacy to Self-Advocacy:** Learner independence integrates key (meta) cognitive strategies and subject content knowledge with a focus on creating thinkers; problem solvers; and self-regulated, life-long learners.

SUPPLIES AND MATERIALS

- Video: *Expressions and Equations: Physical and Mathematical Modeling Part 2– Algebraic Expressions*
  - This video is for NYS MEP use only.
  - Use the video link on the NYS migrant website in the Professional Development section for this workshop, *"Expressions and Equations: Physical and Mathematical*

Modeling Part 2 – Algebraic Expressions"

- Participant Handouts
  1. Power Point Notes
  2. Algebra Tiles: Positive Blue, Green, Yellow
     - Color copy on white cardstock.
     - Cut apart before the workshop or add time for participants to cut apart the tiles.
  3. Algebra Tiles: Negative (red)
     - Copy on red cardstock
     - Cut apart before the workshop or add time for participants to cut apart the tiles
  4. T-Charts
  5. Try Me! Answer Key (Print copy optional)
  6. Website Links Using Algebra Tiles (electronic instead of printed copy)
- Scrap paper for work, writing utensil
- M-TASC Sign-in Sheet (for group participation)
- M-TASC Exit Survey/Evaluation

GETTING STARTED

- Disseminate handouts
  - Power Point Notes
  - Algebra Tiles: Positive Blue, Green, Yellow
  - Algebra Tiles: Negative (red)
  - T-Charts
- Begin video *Expressions and Equations: Physical and Mathematical Modeling Part 2 - Algebraic Expressions*

The video presentation is 26 minutes. Add 15 minutes to stop for discussion and to solve the practice problems at the end.

INTRODUCTION *(2 minutes)*

- Learning Targets
- Algebra Tiles linking the conceptual to the concrete

PREREQUISITS FOR STUDENTS TO WORK WITH ALGEBRAIC EQUATIONS, USING ALGEBRA TILES *(5 minutes)*

Facilitator Note: This section moves fast, reviewing in five minutes what the first module uses 40 minutes to model, practice and explain.

1. Terms in an expression
2. Inverses and making zero pairs
3. The relationship of negatives, parentheses, and inverses

Activity 1: Understanding relationship of negatives, parentheses and inverses

- Participants solve terms using the parentheses.

Key Points

- The key is to work with inverses from inside of the parentheses to the outside.
- Understand that the negative sign can indicate the inverse or opposite of a signed number.

Facilitator Notes: Music plays to give participants time for independent practice. Solutions are explained when the music stops. The video can be paused when participants need additional time.

Activity 2: Conceptualizing addition and subtraction of integers

- Participants use the t-chart handout and integer tiles to model these expressions, find zero pairs and simplify.
- The square tiles marked with a value of 1 are the integer tiles.
  - $-2 + 4 - 1$
  - $3 - 5 + -2$

THE TILES: USING VARIABLES *(10 minutes)*

What does an "X" look like?

- Variables are formally introduced in Grade 6.
- Standard 6.EE.2 – Write, read and evaluate expressions in which letters stand for numbers.
- Standard 6.EE.4 – Identify when two expressions are equivalent.

Key Points

- Using math symbols in an algebraic expression, it is easy for students to confuse the X terms with the X^2 terms.
- When students use Algebra Tiles, it makes sense that the terms are different because the X^2 tile demonstrates the relationship between the two terms.

Activity 3: Modeling operations and polynomials

- Participants lay out the same tiles as shown in the video.
- Write the algebraic expression these represent using symbolic notation.

Facilitator Note: The Power Point Notes handout for this module is more information than a copy of the video's power point. Details about the activity tasks and some solutions were added to help participants process the information.

Activity 4: Modeling operations and polynomials with a T-chart

- Use the same tiles from Activity 3 and place them on a T-chart.
- Find and remove zero pairs to simplify.

Key Points

- Using the tiles in a T-chart allows students to see potential for inverses and zero pairs.
- When students simplify an algebraic expression, they are creating an equivalent expression.

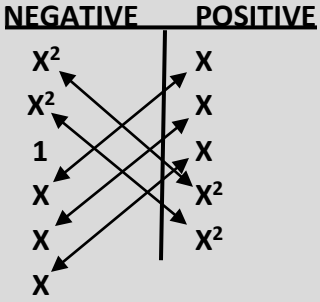
Activity 5: Physical Model to Mathematical Model

- Find the mathematical model based on the physical model on the video.
- Is there a simplified form?

Activity 6: Mathematical Model to Physical Model

- Use tiles to build a physical model of the mathematical expression

Facilitator Note: If participants could use more time to process this solution, pause the video and ask a volunteer to demonstrate how the tiles would look on a T-chart before and after removing zero pairs.



Simplify:

Take out the Zero Pairs: $-X^2 + X^2 = 0$

$-X^2 + X^2 = 0$

$-X + X = 0$

$-X + X = 0$

$-X + X = 0$

The expression simplifies to –1

THE DISTRIBUTIVE PROPERTY *(4 minutes)*

Key Point

- Students in Grade 4 use the distributive property using multiplication over addition in expanded form: 3 (2 + 1)

Activity 7: Modeling the multiplication of quantities

- Model the expression with tiles.
  - The video shows the tiles needed to represent the quantity inside the parentheses.
- Write the simplified expression.

DEMONSTRATION – Using T-charts to build relational understanding

Key Points

- Helps students transition from using the tiles.
- Students can use T-charts as a strategy in Grade 6 and up.

DEMONSTRATION – Using T-charts with the Distributive Property

Key Point

- Use the Grade 3 skill to look for the number of groups and the number/quantity in each group.

INVERSES AND SUBTRACTING QUANTITIES *(2 minutes)*

DEMONSTRATION – Applying the Inverse in the Distributive Property

Key Points

- Expanding student understanding of inverses.
- Starting students with Algebra Tiles and T-charts forces them to pay attention to how the inverse flips the quantity from negative to positive or from positive to negative for ALL the quantities inside the parentheses.

Facilitator Note: Suzanne models the process with tiles alone first, the repeats using a T-chart.

Process

- Model the quantity inside the parentheses.
- Distribute the "**–**"/inverse.
- Model how the colors and quantities change or "flip" from negative to positive; or from positive to negative.

SUMMARY AND CLOSING *(3 minutes)*

DEMONSTRATON – Putting it all together

Activity 8: Try Me! optional practice using T-charts to simplify algebraic expressions

Facilitator Note: You can pause the video for participants to simplify the expressions or let the presentation finish before solving.

Expressions: Participants have the expressions in the Power Point Notes.

5X – (3X – 4) **(5X – 1) - (3X – 4)**

2 + X – (X – 3) **(5X – 1) – (3X – 4)**

Facilitator Note: Printing the *Try Me! Answer Key* handout is optional. You can distribute to participants so they can self-check or you can project the handout on the screen for participants to self-check. Discuss the process to reach the answers as needed.

FOLLOW UP ACTIVITY – RECOMMENDED WEBSITES

Facilitator Note: You can ask volunteers to test the websites, then report to the rest of the staff at the next in-service. This can build in a review about using the Algebra Tiles and leverage the tech-adventurers in the group. Emailing the Website handout to interested staff lets them use the links.

Website: Algebra 4 All	
Algebra Tile Applet	http://a4a.learnport.org/page/algebra-tiles

- Interactive
- Can use online or off

How to download and save to flash drive for use off-line

- On the website, scroll below the Applet and click on the link to download the applet to use off-line. (It is designed to save to a flash drive.)
- In top right of screen, click on the download arrow.
- See the file name: MVU-AlgebraTiles.exe.
- To the right is a square-like graphic – click on this.
- Now you can copy and paste the file to a flash drive.

Using the Applet

- Use the mouse to drag tiles.
- Can switch between types of practice.
- It is worthwhile to start with the introduction to see how it works.

Website: Math Bits	
Working with Algebra Tiles	https://mathbits.com/MathBits/AlgebraTiles/AlgebraTiles/AlgebraTiles.html

- Interactive
- Only online use

Navigating around "Working with Algebra Tiles"

- Use the [PREVIOUS] or [NEXT] buttons to move ahead or back.
- Use the [TOC] button to switch to the Table of Contents.
- At the Table of Contents [TOC], you can click on the underlined topic you want to practice.

Activities

- Let's Start Using Our Tiles
  - Using the positive and negative integer tiles [1] to practice various operations
- Working with Polynomials
  - Using all the tiles and operations with variables

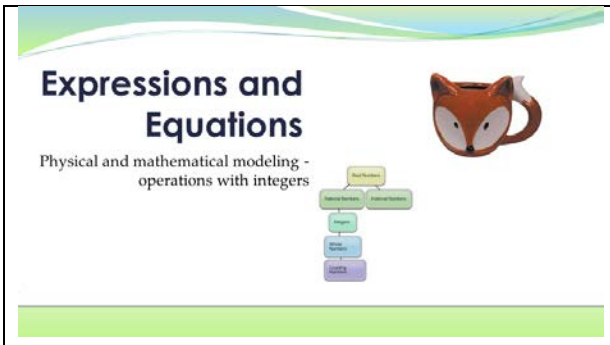
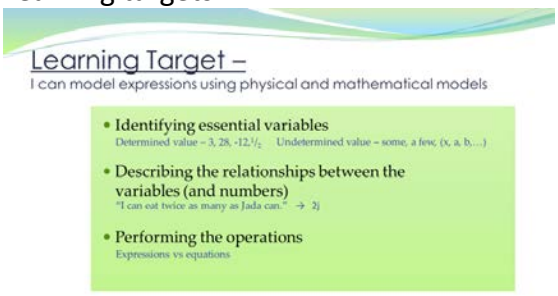
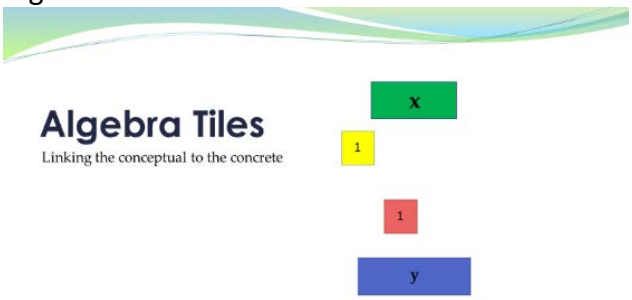
Closure for Group Workshops

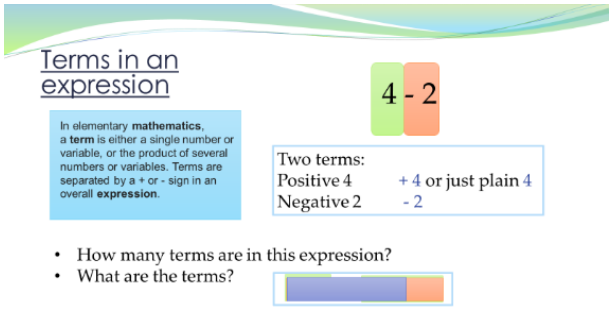
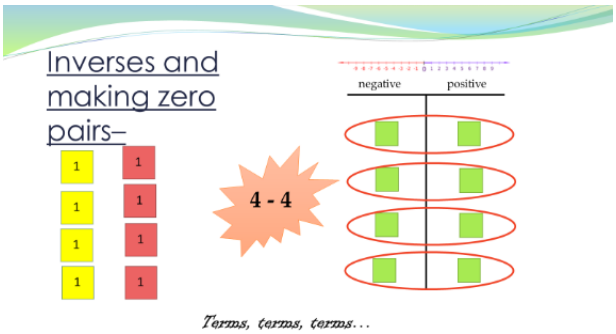
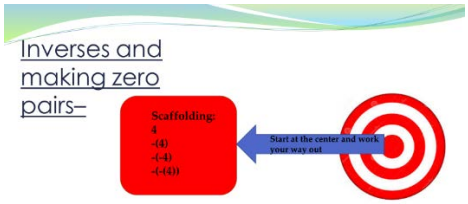
- Facilitators are welcome to use the Workshop Evaluation provided on the website or you can use your own version.
- *Continuing Teacher Leader Education (CTLE)* – Follow the CTLE process at your METS program center for staff who are tracking credit.

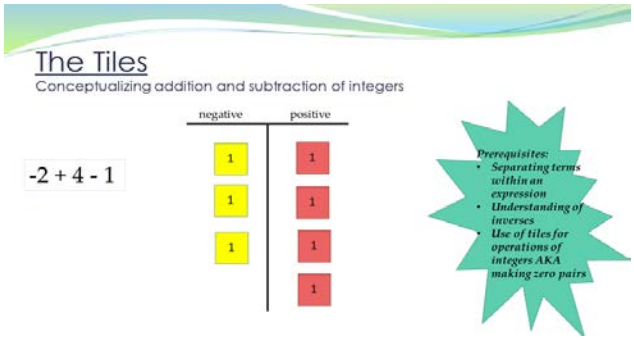
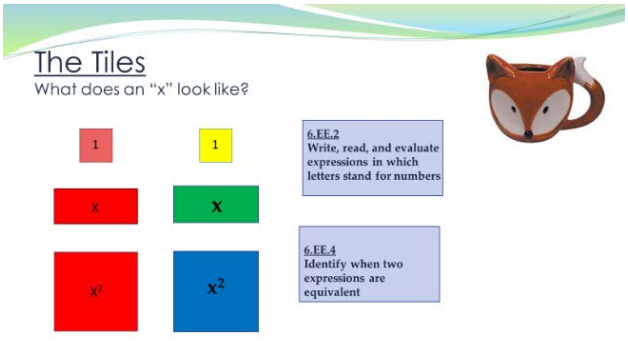
Closure for Individual Access/Self-Serve

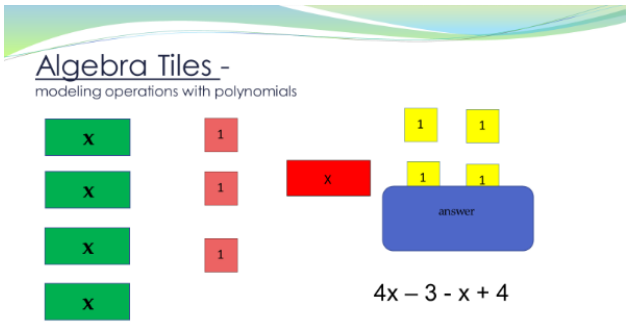
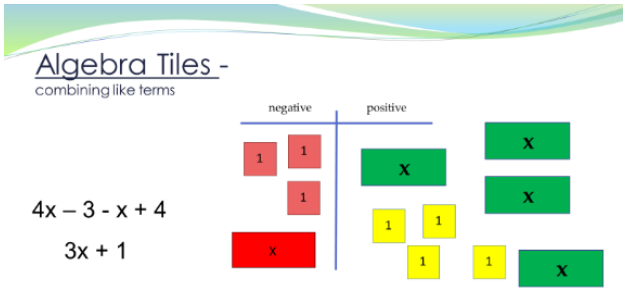
- Complete the Workshop Evaluation and give it to your Director.
- *Continuing Teacher Leader Education (CTLE)* – If you would like to request credit for this module, please follow the CTLE Credit Request process. Find the link for this process on the NYS-MEP website: https://www.nysmigrant.org/resources/pd

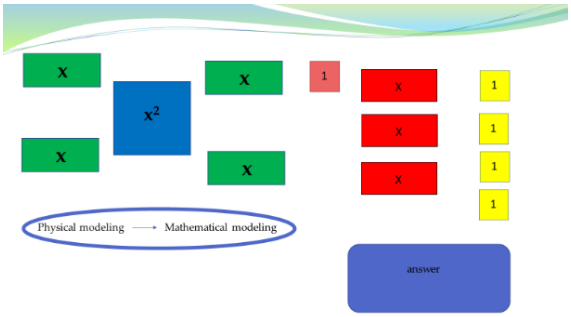
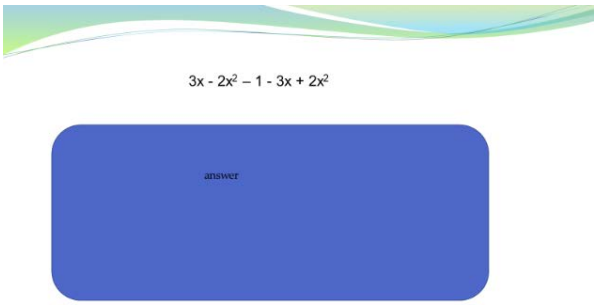
Facilitator Note: The following Appendix contains the workshop "Talking Points" used by Developer, Suzanne K. Fox, to support your facilitation when participants need something repeated.

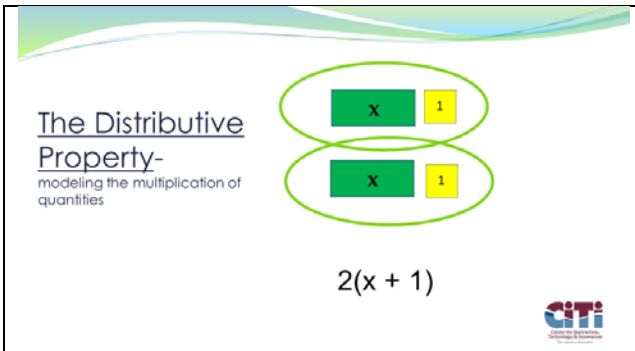
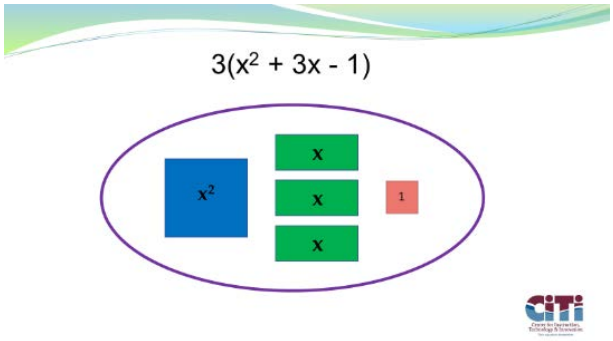
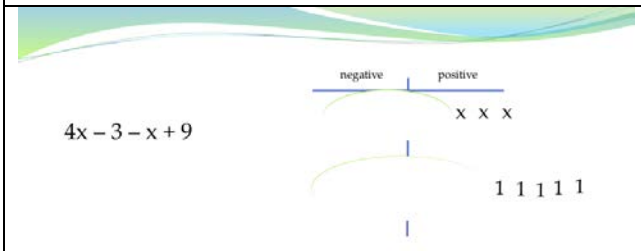
	Today's web learning session will allow you to model algebraic expressions using algebra tiles and T-charts. It will enable you and your students to understand how variables behave in the same relationships with each other as do numbers without using traditional "rules". This ability to have a deeper understanding of algebraic thinking will assist students as they transition from working with the concrete to the abstract in mathematics. For this learning session you will need your set of algebra tiles, the handout with the T-chart templates and some paper to work out some algebra problems. If you are not an expert in algebra, you will be well on your way after this video!
Learning targets 	Overall this series has three learning targets. In this second segment, our three learning targets expand to include variables, along with integers. These same learning targets help students move to algebraic thinking and using variables and determined values.
Algebra tiles 	Algebra tiles are an excellent means to bring the symbolic structure of integers and variables to a concrete level. Using the tiles, we will be able to model algebraic expressions so students can "see" how variables relate to each other. Many students struggle with moving from the concrete number world to the abstract, symbolic language of algebra. Physical models can help bridge this transition as you soon will find out through our exploration.

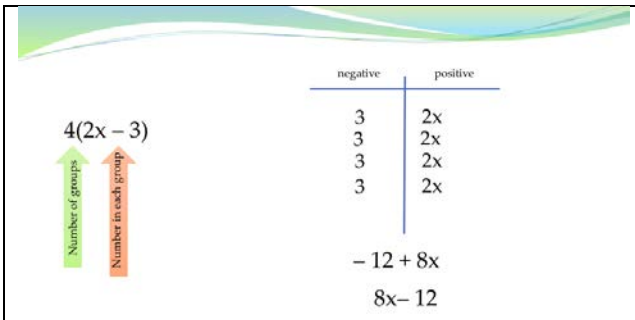
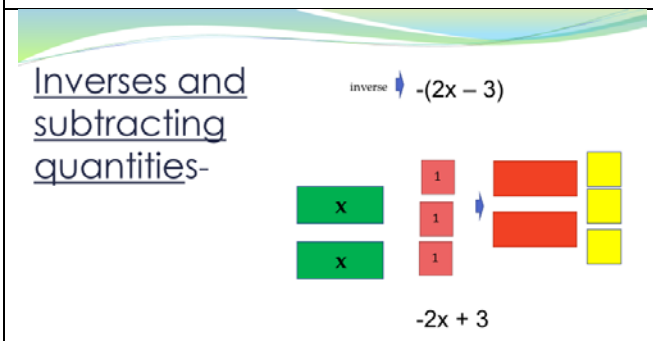
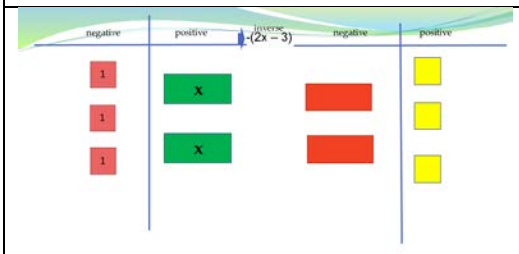
Terms in an expression 	Let's quickly review the three prerequisites students need in order to work with algebraic expressions involving algebra tiles. The first is knowing about the "terms" in an expression. In the example of 4 minus 2 there are actually two terms. The first term CLICK is positive 4 and the second term is CLICK negative 2. CLICK Take a look at the expression in the blue box. Students need to be able to answer the following questions when given an algebraic expression. How many terms are in this expression? What are the terms in this expression? In this case, there are four terms. The first CLICK is 14 CLICK the second is CLICK CLICK the third is CLICK CLICK and the fourth is CLICK 9. CLICK
	The second prerequisite is the concept of inverses and making zero pairs. Start out with a simple example such as 4 – 4 Using the yellow tiles as positive and the red tiles as negative, this expression is modeled on the left side of the slide. Using a 1 to 1 correspondence the answer is zero. CLICK 4X Using the t chart, the same 1 to 1 correspondence happens to model the expression. The chart itself indicates which is positive and which is negative. By circling the zero pairs it is shown how 4 minus 4 is 0. 4 CLICKS
	The third prerequisite is the relationship of inverses. The key is always to work from the inside out in terms of the parentheses. Take a moment to jot down what the final outcome of each of these four number problems would be. MUSIC

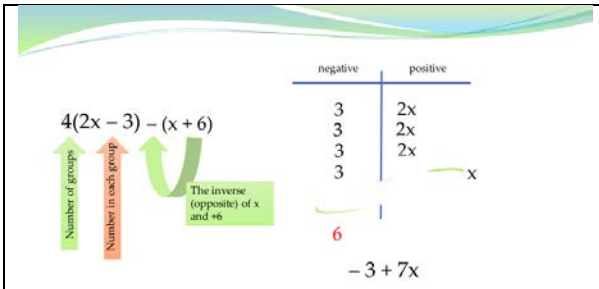
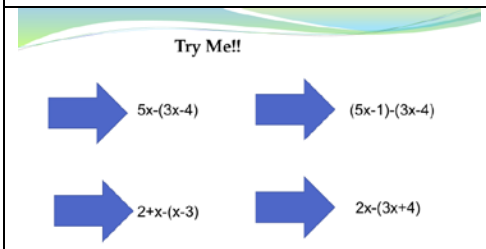
	The first is positive 4. The second means take the opposite (or inverse) of positive 4. That is negative 4. The third means the inverse of negative 4, which is positive 4. The fourth means the inverse of the inverse of 4 so we are right back at positive 4. The student's goal is to understand that inverse of a number is the same distance from zero on a number line in the opposite direction.
	Let's prep for our work with variables doing a few integer problems. Try the two on this slide. You can use your tiles or a T-chart. MISIC Conceptual understanding for success hinges on the three conceptual foundations of: Looking at terms within an expression Understanding inverses Making zero pairs. Time to check our work! Negative 2 positive four and negative one simplify to positive one. Positive 3 negative 5 and negative two combine to make negative 4. Now let's move on to variables!
	Students who struggle with the transition from the concrete to abstract can use algebra tiles to develop the algebraic/number sense in realizing that the properties of defined/determined values are used in exactly the same way with variables. Grade 6 is when the formal concept of a variable is introduced. These two standards are critical foundations for basically all future understandings of algebra. They deserve our full attention and so we will start here using our tiles.

	Let's start building some algebraic expressions using tiles. Knowing the three prerequisites will help your students from asking the question that I heard so many times as a teacher… Mrs. Fox is that a minus sign or a negative sign? My answer would always be… What would you like it to be? Why answer that way…it all depends on if one is looking at the entire expression, or examining the terms! Take your tiles and lay them out so they match the ones on the slide. Then see if you can write what this algebraic expression would be using symbolic notation. MUSIC Click to reveal the answer. Did you have this expression?
	You may not have! Take a look at the same expression, and this time put into a T-chart. Using a t chart allows students to see the potential of inverses, of making zero pairs! Take your tiles, put them in your t chart, and make some zero pairs. What do you end up with? MUSIC If you were successful at making zero pairs, the expression of 4X – 3 – X + 4 can also be written as 3X + 1. What you actually did is called simplifying an algebraic expression or creating an equivalent expression. And all you did was use your three prerequisites along with some fancy tile moves!

	What would be the mathematical model based on this physical model? Is there a simplified form for this model? Work independently or with a partner to find out. MUSIC CLICK TO REVEAL One of the most frequent mistakes students make in simplifying expressions is combining (when they should NOT be combining) variables and variables with exponents. CLICK When given just the mathematical model, students try to combine ALL Xs regardless of the exponent. When the tiles are used to model this physically, students really see the difference between an x and an x squared. Having that simple, yet hugely important deeper understanding eliminates the urge to just combine it all!
	Now let's do a challenge and work backwards. Given the mathematical model of the expression, can you use the tiles to create a physical model? MUSIC CLICK How many of you saw some opportunity to simplify the expression and did so already? The key here is that when you use the tiles with your students, they will be showing YOU when there is opportunity to simplify. They will be internalizing the strategy to always look for ways to simplify. Being able to look at ways to manipulate algebraic expressions is key for future learning in upper mathematics.

	Did you know that students in Grade 4 understand the distributive property? They know it from using multiplication over addition in expanded form! The quantity of X plus 1 can be considered an expanded form. Why? Because we can't combine anything until we know what X is! So by looking at this slide can you see that modeling the expression 2 times the quantity of X + 1 is making two groups of X + 1 and then just combining like terms/tiles.
	Once your students understand how the distributive property works on both defined numbers and variables, you can go nuts with some really long quantities! Try modeling this example with a partner using your tiles. Then write your simplified expression symbolically. MUSIC CLICK How confident were you with your answer using the tiles? Do you feel confident moving away from the tiles to a T-chart? Let's use a t chart to show how expressions can be modeled to help build relational understanding in algebra.
	In order to simplify the expression, we look at the terms within the expression. The first, 4X, means that there are four positive Xs. CLICK. Then the second term is negative 3 so we put three on the negative side CLICK. The third term is one negative X CLICK and the fourth term is positive 9 CLICK. Now we see about making some zero pairs. We have Xs CLICK CLICK and three units CLICK we can circle and eliminate. CLICK. What is left is the simplified answer CLICK

	We can use the T-chart for the distributive property as well. Remembering our thinking process CLICK of number of equal groups gives the ability to just write the quantity CLICK four times in the chart and tally up the final answer. CLICK This is also a great time to discuss how this expression can be written in two ways CLICK by using the commutative property and our understanding of terms in an expression.
	Our final step in working with algebraic expressions is working with inverses in subtracting quantities. In this case and the examples that follow, it is just expanding on our understanding of the inverse of a number or term. So, when we take the inverse of terms in parentheses, all we are doing is taking the inverse of each term inside. I would highly recommend using algebra tiles and the T-chart for this at the start, as I know from experience in the classroom, that students often forget to take the inverse of EVERY term. Using tiles and the T-chart helps to cement the practice of inverses for all terms in the quantity. The example of taking the inverse of 2X – 3 using the tiles shows how each term is “flipped” from positive to negative and negative to positive using the colors.
	Here is the same example using the T-chart. Notice I am really stressing the inverse by still using the colors. It is well worth the time to do this as once students really understand the inverse all subsequent expressions just follow the same procedure.

	Let's put all that we have discovered through this learning experience in one final example. I hope you recognize the first part as we showed it in an earlier slide. Putting the rest CLICK of the expression in makes the problem look harder, however it is just putting a few more items in the T-chart. The inverse of a quantity CLICK means to take the inverse of each term inside the parentheses. So, the inverse of X is negative CLICK X and the inverse of positive 6 is CLICK is negative 6. In order to make zero pairs, one of the 2Xs needs to be broken down CLICK and then we can eliminate one of the Xs in a zero pair. CLICK All that remains is to tally up the columns in the t chart. CLICK
	If you would like more practice in this area, here are several examples. Your facilitator has the answer key for all four using the T-chart. Feel free to try one or all to sharpen your skills and then try them out with your students.
	It is so important to bridge the transition from working with numbers to working with variables using both physical and mathematical models. If you think about it, that is the way it happens in our lives. We see a problem, think about it, map out a strategy on paper, and then work to arrive at a solution. We are teaching much more than math by doing this process. We are creating problem solvers. Please feel free to explore the websites on this slide to deepen your and your student's understanding of both physical and mathematical modeling in algebra. There are tutorials and practice sheets within both these sites. Thank you again for allowing me to be part of your math journey.