NEW YORK STATE MIGRANT EDUCATION PROGRAM

Title: Expressions and Equations: Physical and Mathematical Modeling
Part 1 - Operations with Integers

Description: This module is an approximately 40-minute presentation on modeling operations with integers using algebra tiles and T-charts. Students will gain conceptual understanding of addition and subtraction of negatives and positives working from the concrete to the symbolic. The outcome will be a deeper understanding of integers and foundational work for operations with algebraic expressions and equations. Intended audience: tutors, teachers.

Developer: Suzanne K. Fox, Staff Development Specialist, Oswego Center for Instruction, Technology & Innovation (CiTi)

Series: This is the first in a two-part series on Expressions and Equations using Algebra Tiles for the physical and mathematical modeling.

- Part One – Operations with Integers
- Part Two – Algebraic Expressions

Facilitator Guide

INDIVIDUAL ACCESS/SELF-SERVE *(for Individuals viewing this module independently):* While a robust conversation between colleagues is an enriching way to learn, so is self-reflection. Read and use this Guide as the Facilitator of your own learning. To get the most out of the activities and questions, make sure you have the recommended handouts and supplies listed below, before beginning.

CTLE CREDIT

Group Workshop: If you are facilitating this workshop for your METS, you will have to decide which process you will use for granting CTLE credit. You can use your local LEA process, or the M-TASC process:

a. Contact M-TASC in advance of the workshop to confirm date and module.
b. Use the M-TASC Participant Sign-In Sheet and submit.
c. Submit Workshop Evaluations via link or hard copy. If you use the Evaluation link, M-TASC will forward the compiled evaluations once you have informed the office that all evaluations are complete.

Individual Access/Self-Serve: For those who would like to request Continuing Teacher Leader Education (CTLE) credit for On-Demand professional development, please complete the CTLE Credit Request for each module. Find the link for this process on the NYS-MEP website: https://www.nysmigrant.org/resources/pd

OBJECTIVES/LEARNING TARGET(S)

- I can use visual models to show the relationship of positive numbers to negatives using zero pairs.
- I can model both physically and mathematically addition and subtraction with integers.

WORKSHOP/MODULE DESIGN

This web-learning session will allow you to model operations with integers using algebra tiles and t charts. It will enable you and your students to understand how negatives and positives are used with numbers without using traditional "rules". This ability to have a deeper understanding of integers will not only help when working with rational numbers, it will continue to work with operations with algebraic expressions and equations. The target audience for this workshop are educators working in grades 6-8 where the emphasis is on conceptual and procedural understanding of operations with both integers and rational numbers. This is the first in a two-part series on using the Concrete, Representational, Abstract (CRA) approach to understanding operations with negative numbers.

CONNECTION TO THE NYS MEP THEORY OF ACTION

- New York State Migrant Education Program Theory of Action
  - **Subject Content and Instruction Subject:** Focus on assuring that in-school students the foundational skills and strategies to succeed in the classroom and on state and other assessments.
  - **Advocacy to Self-Advocacy:** Learner independence integrates key (Meta) cognitive strategies and subject content knowledge with a focus on creating thinkers; problem solvers; and self-regulated, life-long learners.

SUPPLIES AND MATERIALS

- Video: *Expressions and Equations: Physical and mathematical modeling Part 1 – Operations with Integers*
  - This video is for NYS MEP use only.
  - Use the video link on the NYS migrant website in the Professional Development section for this workshop, " *Expressions and Equations: Physical and Mathematical Modeling Part 1 – Operations with Integers*

- Participant Handouts
  1. Integer Tiles: Positive and Negative
     - Copy in color
     - Cut apart before the workshop
  2. T-Charts
  3. Integer Problems – Practice A-B-C
  4. Grade Emphasis Table with Standards for Grades 3-8
     - Save for reference *(printing optional)*
- M-TASC Sign-in Sheet
- M-TASC Exit Survey/Evaluation

GETTING STARTED

- Disseminate handouts.
- Begin video – Expressions and Equations, Working with Integers.

Total time for video is 40 minutes. Allow 5 additional minutes to pause the video and complete the last activity.

Facilitator Note: During the video, participants are asked to use the algebra tiles and handouts for activities.

INTRODUCTION (3 minutes)

- Learning Targets
- NYS Standards and Emphasis – the roadmap of the grade
- Algebra Tiles

THE ZERO EFFECT (10 minutes)

Key Points

- NYS Learning standard 6.NS.6a – Recognizing opposite signs of numbers as indicating location on opposite sides of 0 (zero).
- To understand the relationship of negatives to positives, need to know about *terms* in an expression.
- A term is either a single number or the product of several numbers or variables.
- Terms are separated by a plus (+) or minus (–) sign in an overall expression.
- Understand the concept of numbers being equal distance from zero are inverses and when combined "cancel" each other out to become zero.

Activity 1: Introducing the tiles

- Presenter Suzanne Fox demonstrates the use of the positive and negative tiles with a T-chart.

Facilitator Note: There is a brief suggestion for participants to practice with the tiles, but the video moves right into the next demonstration. You can pause the video for this initial practice, but it might be more important to pause the video when there is a more difficult concept later in the video.

Activity 2: Recognizing terms in a numeric expression

- Need: Pre-cut algebra tiles (red and yellow), T-Chart handout
- Independent Activity: Participants identify the terms in an expression, while the music plays
- After the music, Suzanne reviews the results with the group

Activity 3: Making zero pairs

- Independent Activity: making zero pairs
- Independent Activity: using the T-Chart to make zero pairs

Facilitator Note: The video will play background music while participants work (sometimes in pairs) to complete the activities within the presentation. If the music stops before the group is ready, please feel free to pause the video. Conversely, if participants struggle, know that the music plays for a maximum of 1 minute so that explanations can be modeled in a timely manner.

ADDITION AND SUBTRACTION OF INTEGERS (27 minutes)

Key Points

- Perform addition and subtraction of integers with algebra tiles using the concepts of recognizing terms and zero pairs
- The relationship of negatives, parentheses and inverses
- Understand that the negative sign can indicate the inverse or opposite of a signed number
- Perform subtraction and addition of integers with algebra tiles using the concepts of applying inverses, recognizing terms, and zero pairs

Activity 4: Simple addition and subtraction of integers

- "Practice **A**" on handout, Integer Problems - Practice A-B-C.
- Group Activity: Suzanne models the addition and subtraction with the algebra tiles and a T-Chart.

Activity 5: The relationship of negatives, parentheses and inverses

- Independent Activity: Using the expression on the video, identify the *terms.*
- Independent Activity: Finding the opposites/ the inverses of the numbers in the green box.

Activity 6: Subtraction of integers and inverses

- "Practice **B**" on handout, Integer Problems - Practice A-B-C
- Presenter Suzanne will model how the first expression in Practice set B.
- Independent Activity: Participants model each expression, one at a time on their own or with a partner during the music. Then Suzanne provides an explanation.
    - Repeat this pattern through the expressions in Practice B.

Activity 7: Complex addition and subtraction of integers

- "Practice **C**" on handout, Integer Problems - Practice A-B-C
- Independent Practice: Pause the video when directed for participants to model all of the expressions in Practice Set C.
- Return to the video for Suzanne's explanation, including more ways to vary the modeling to meet the complexity and student needs.

Facilitator Note: The video identifies two (2) Website Resources for Expressions and Equations. The first website resource is free, but has many ads on the site to avoid.

- Ignore the multiple ads for "XL – 10 FREE practice problems." (This is an AD and isn't free.)
- The blue rectangles to the left and blue links belong to this site.

http://www.onlinemathlearning.com/adding-integers-2.html

The second resource is a PDF.

- Pages 10-16 have examples of using the Zero Pair concept with the full set of Algebra Tiles, which include tiles to represent, X and X^2!
- The second part of this series, Expressions and Equations: Physical and mathematical modeling Part 2 – Algebraic Expressions, models this and more with the full set of Algebra Tiles.

https://dccmiddle.asd20.org/Teachers/Susan_Turner/Documents/algebra%20tiles%20workbook.pdf

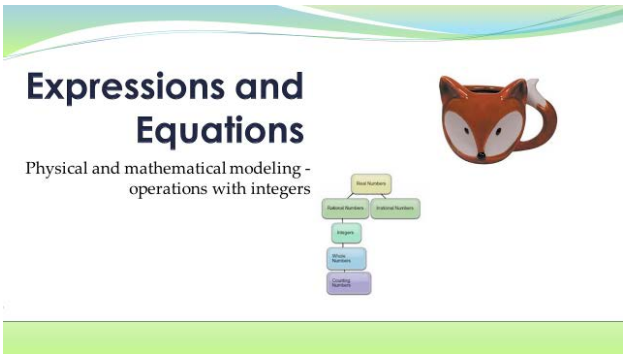
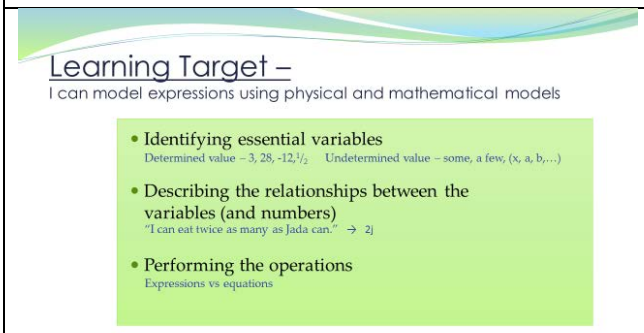
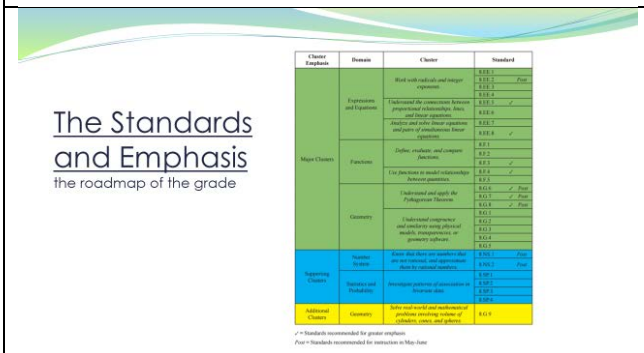
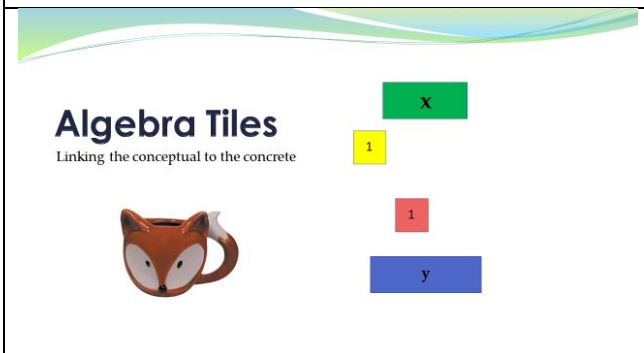
Closure for Group Workshops

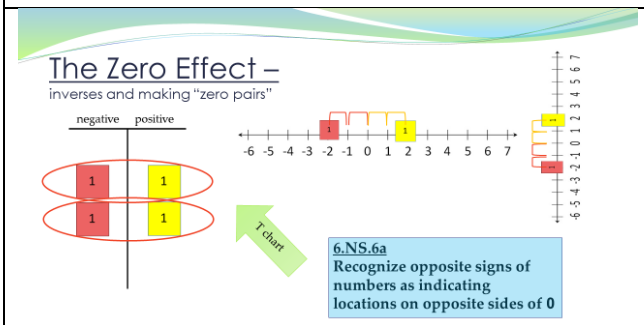
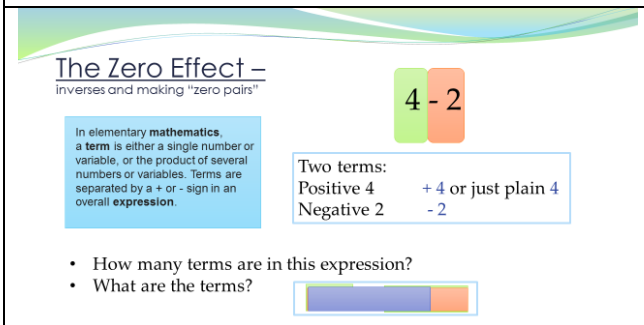
- Facilitators are welcome to use the Workshop Evaluation provided on the website or you can use your own version.
- *Continuing Teacher Leader Education (CTLE)* – Follow the CTLE process at your METS program center for staff who are tracking credit.

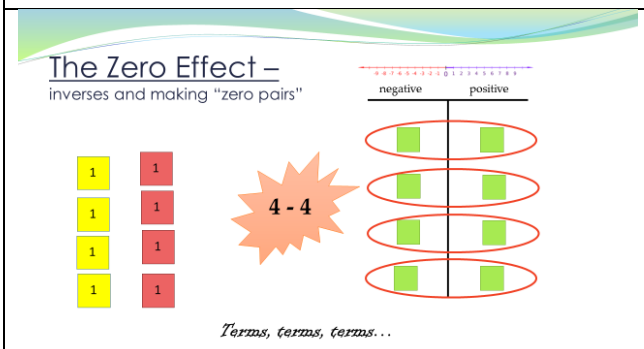
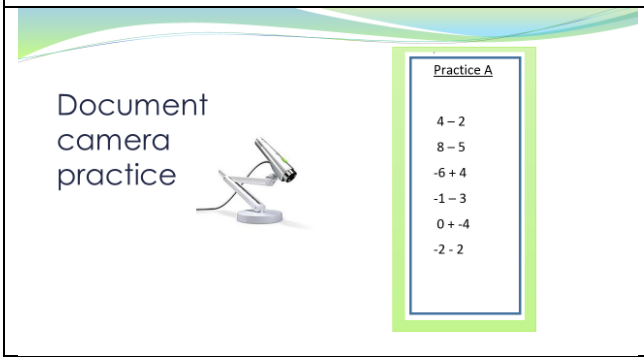
Closure for Individual Access/Self-Serve

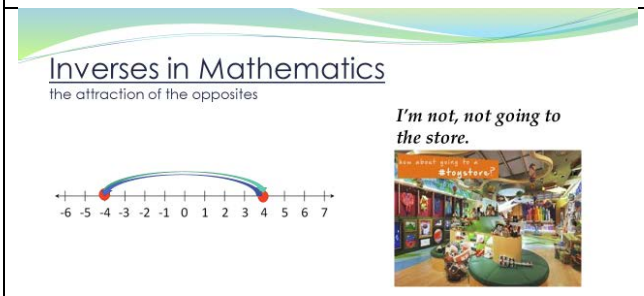
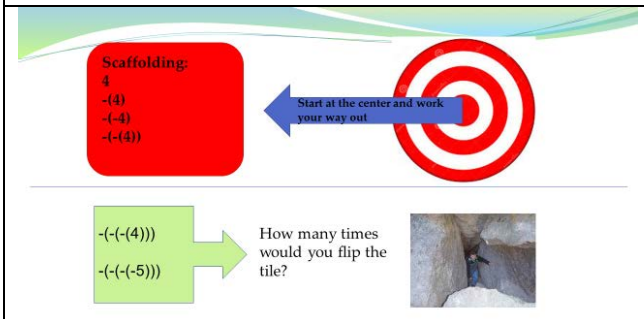
- Complete the Workshop Evaluation and give it to your Director.
- *Continuing Teacher Leader Education (CTLE)* – If you would like to request credit for this module, please follow the CTLE Credit Request process. Find the link for this process on the NYS-MEP website: https://www.nysmigrant.org/resources/pd

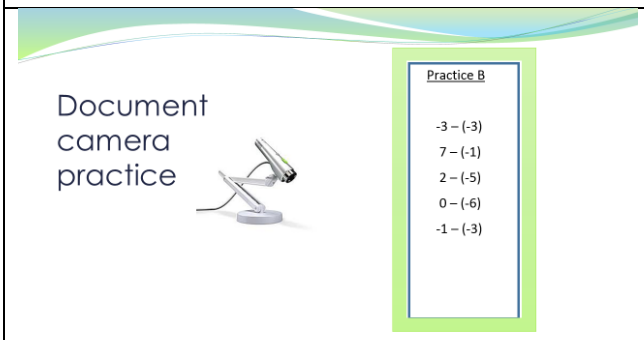
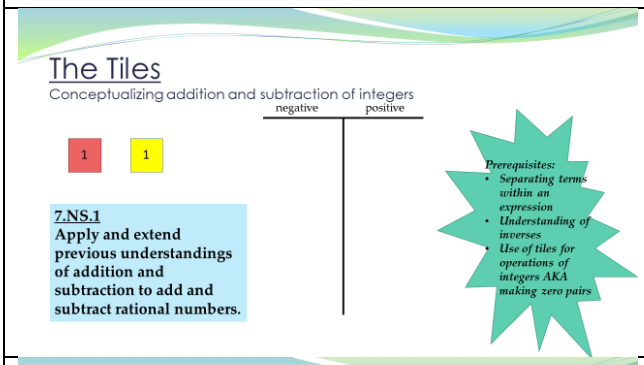
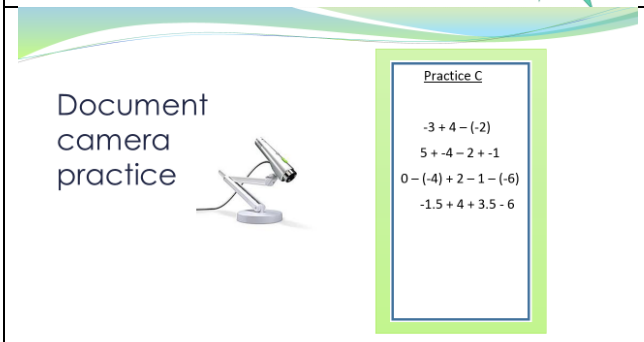
Facilitator Note: The following Appendix contains the workshop "Talking Points" used by Developer, Suzanne K. Fox, to support your facilitation when participants need something repeated.

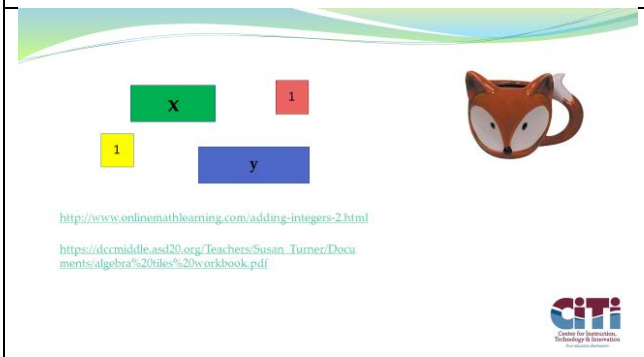
	Today's web learning session will allow you to model operations with integers using algebra tiles and T charts. It will enable you and your students to understand how negatives and positives are used with numbers without using traditional "rules". This ability to have a deeper understanding of integers will not only help when working with rational numbers, it will continue to work with operations with algebraic expressions and equations. For this learning session you will need your integer tiles, both the yellow and red. You will also need the handouts with the T chart templates and integer problems. Finally, you will need the standards emphasis chart from the grade level you are currently working with.
	Overall this series has three learning targets. In this first segment, our three learning targets are confined to the set of rational numbers, with emphasis on the set of integers. These same learning targets are used when moving in to algebraic thinking and using variables in place of determined values.
	The standards and emphasis chart is from the educators guide to the NY state exam. Here you will see the major, supporting, and additional clusters and standards that are taught at each grade level. Standards with a check are ones that should be given particular attention to. Some people refer these to the power standards of the grade. When you are working with your students, and need to decide where to focus your attention, this document is an excellent place to start.
	Algebra tiles are an excellent means to bring the symbolic structure of integers and variables to a concrete level. Using the tiles, we will be able to demonstrate the integer rules of addition and subtraction, as well as the concept of an inverse. Understanding how negatives act on the value of a number can make memorizing rules unnecessary. This understanding then carries to a

	much greater understanding of how negatives work with variables. Many think that manipulatives are best for lower grades. While we are working with them in this session, try to be aware how working with these tiles enhance the understanding of this complex concept.
	The first step in developing a conceptual understanding of negative numbers begins in grade 6. A number line is an excellent tool to show the relationship between positive and negative numbers and 0. CLICK SLOWLY 4 TIMES Using a number line your student can physically count how many spaces till zero on the left side and then see that the same number of spaces is on the right side. This is the first step in understanding the concept of "zero pairs" and eventually inverses. CLICK Turning the number line on its side really makes it a thermometer and students may understand that negatives are "colder" as they go further away from 0 and positives get warmer. A second way of showing the relationship of negatives and positives is by using CLICK a t chart. Using a t chart has the ability to show the 1 to 1 relationship CLICK 2 between positives and negatives and how making a zero pair is really combining CLICK 2 the numbers to end up with zero as the number. Let's go to our t charts and do some modeling with our tiles. DOCUMENT CAMERA
	The second component to conceptual understanding of the relationship of negatives to positives is knowing about the "terms" in an expression. In this first example of 4 minus 2 there are actually two terms. The first term CLICK is positive 4 and the second term is CLICK negative 2. CLICK Take a look at the expression in the blue box. How many terms are in this expression? What are the terms in this expression? Take a minute with your neighbor to confer and write down your answers.

	MUSIC In this expression, there are four terms. The first CLICK is 14 CLICK the second is CLICK CLICK the third is CLICK CLICK and the fourth is CLICK 9. CLICK Knowing how to separate an expression into its terms allows the ability to work with the negatives and positives in relation to each other.
	Now that we have the two foundational understandings, working with integers is more of a working with this relationship than memorization of rules for the sake of rules. Let's start out with a simple example such as 4 -4. Your student may say "that's so easy, the answer is 0". Well, do they know WHY the answer is zero? Using the yellow tiles as positive and the red tiles as negative, take a moment to model this expression as you see it on this slide. MUSIC Now use the concept of zero pairs in a 1 to 1 correspondence to show that the answer is indeed zero. Now let's try using the t chart to model the same expression. Place your tiles on the t chart as you see on this slide. MUSIC Notice that I used green tiles. When you use the t chart, it does not mandate that specific colors are used as the chart itself indicates which is positive and which is negative. Now you can circle the zero pairs and again physically show how 4 minus 4 is 0.
	Let's move to the document camera and practice using the tiles to model these expressions and determine their value. PARTICIPANTS MODEL THE EXPRESSION, THEN I MODEL AND SHOW HOW TO GET THE ANSWER.

<table>
<tr>
<td> 
</td>
<td>So, when will your students need to use this skill? The actual standard of understanding the concept of negatives is in green. All the standards in red are contingent on the students' ability to have this conceptual understanding. Pretty amazing how math builds on this foundation.</td>
</tr>
<tr>
<td> 
</td>
<td>Once you see that your student has this conceptual understanding, the concept of inverses becomes invaluable as integers are soon to be used with variables and then in algebraic expressions and equations.
Take a moment with a neighbor and have a discussion about the sentence, I'm not not going to the store and how it could be modeled on the number line diagram.
MUSIC
First, I am hoping that you realized that the use of the double negative actually canceled itself out. In other words, I'm not not going to the store means that you ARE going to the store.
The same thought goes into the inverse in math. Let's start with positive 4 CLICKS. The opposite of positive 4 is negative 4, and if I take the opposite of the opposite of positive 4, I am back to where I started! Mathematically speaking, the negative of a negative is a positive.</td>
</tr>
<tr>
<td> 
</td>
<td>Let's take a closer look at this relationship of inverses. The key here is always to work from the inside out in terms of the parentheses. Looking at the red scaffolding –The first is positive 4. The second means take the opposite (or negative) of positive 4. That is negative 4. The third means the opposite of negative 4, which is positive 4. The fourth means the opposite of the opposite of 4 (remember our not not going to the store?) so we are right back at positive 4. You could model this simply by turning a tile or counter over for each negative or opposite.
Work with your neighbor to determine the final outcome for each of the two expressions in the green box.</td>
</tr>
</table>

	MUSIC Once you get the hang of what the opposite or inverse really does to the value of a number, you could really get creative with your student and make this into one challenge after another. Oh….the two answers?... negative 4 and positive 5.
	Let's now take some time to practice using inverses in solving some integer problems. We will move to the document camera now for our work.
	Now with all the basic practice, we can move on to more complex addition and subtraction practice. Remember that moving forward depends on three conceptual foundations: Looking at terms within an expression Understanding inverses Making zero pairs.
	Let's try our top-level practice now. I would ask you to pause the video and work with your neighbor to solve the four problems in row C. Notice that the last problem uses decimals. You can still model this using a t chart. When you are ready to go through the solutions, please start the video again. PAUSE HERE GO TO DOCUMENT CAMERA AND MODEL THE FIRST TWO USING TILES AND THE LAST TWO USING NUMBERS AND THE T CHART

	So where is the connection in math? Remember that our focus standard is in green. The blue represents all the standards that lead up to our green. The red represents all the standards that students are now ready to learn based on their understanding of our green standard. Math is not just a linear progression. It is truly a web of concepts and understandings.
	So I hope you have noticed that I never told you the rules of adding and subtracting integers. Yet you probably are able to solve any integer problem at your students' level. You have just been through a true conceptual understanding (otherwise known as an "AHA" moment)! Please feel free to explore the two links on this final slide. They have many additional examples, modelings, templates and explanations for algebra tiles. If you are motivated to further explore algebra tiles and variables, please ask for the second in this series. It uses tiles for conceptual understanding of algebraic expressions. Thank you so much for having me with you on your math journey.